Baroclinic Instabilities in the Wave-Forced Ocean Mixed Layer

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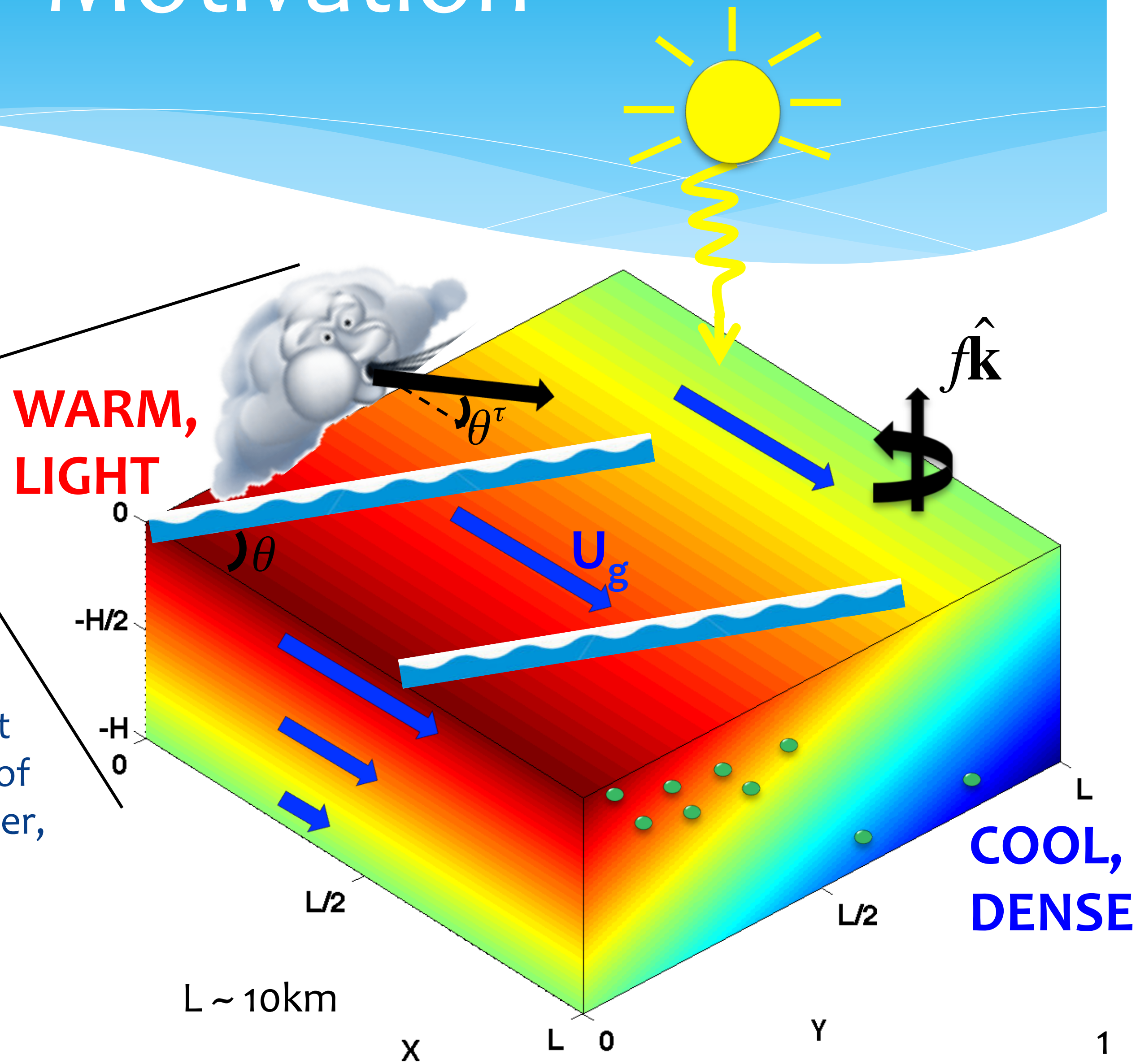
CASPO Seminar
Scripps Institution of Oceanography
February 25th, 2015

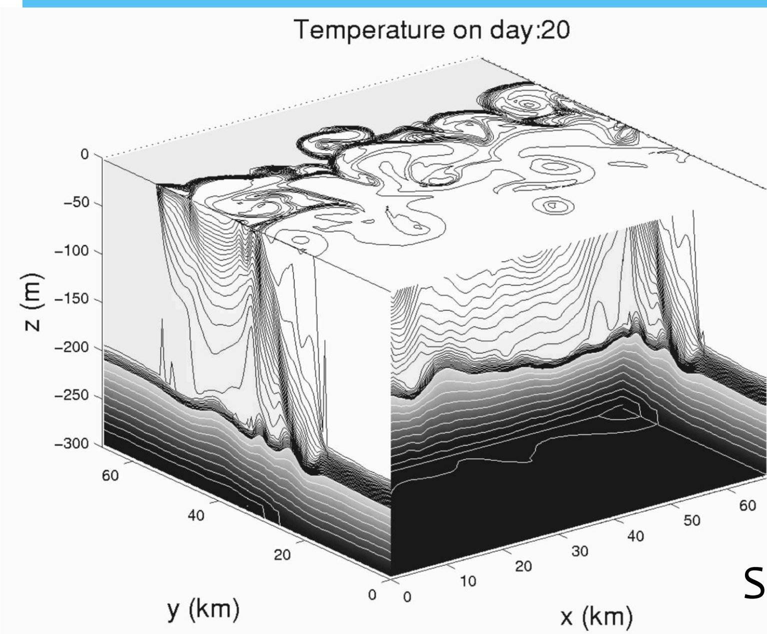
Motivation



* Coarse global models do not accurately represent fluxes of heat, momentum, fresh water, and gases.
* Phytoplankton blooms are influenced by mixed layer dynamcis

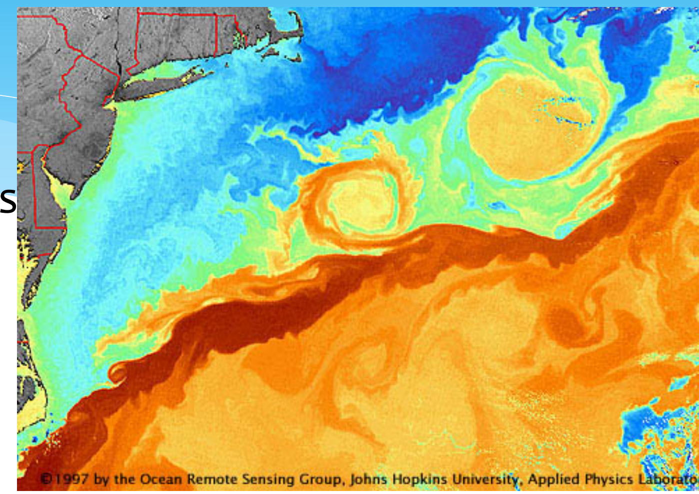
What’s Out There?

Restratifying:



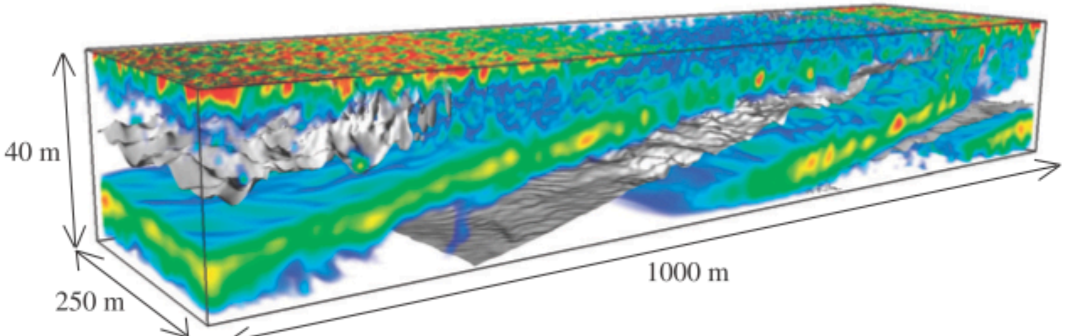
Fox-Kemper and Ferarri 2008

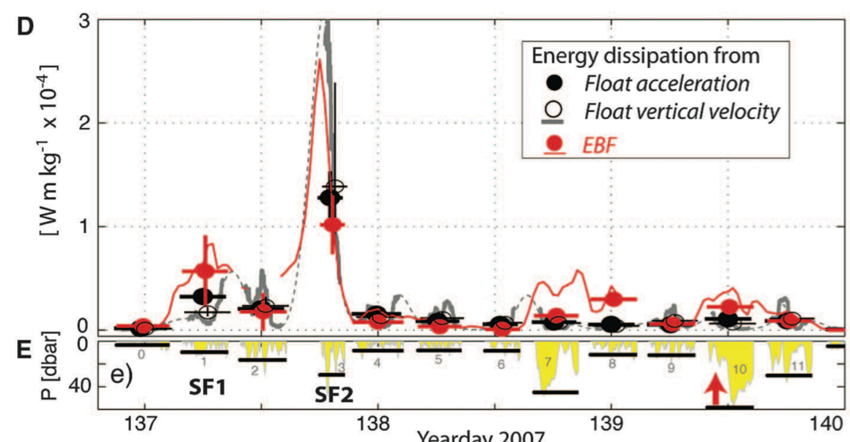
Mixed layer eddies (Geostrophic Instabilities; GI)



Symmetric instabilities (SI)

Taylor and Ferarri 2010



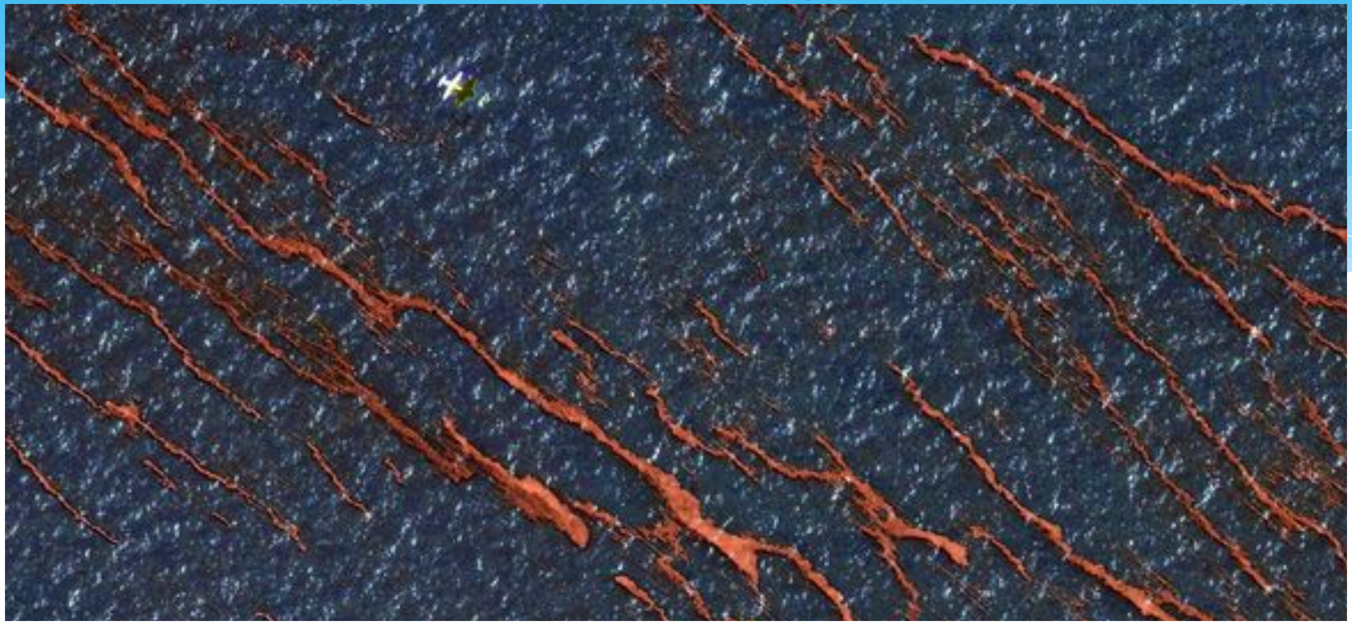


D’Asaro et al., 2011

What's Out There?

Langmuir Circulation/mixing/turbulence (LC)

Deep Water Horizon oil slick. Digital Globe (2010)

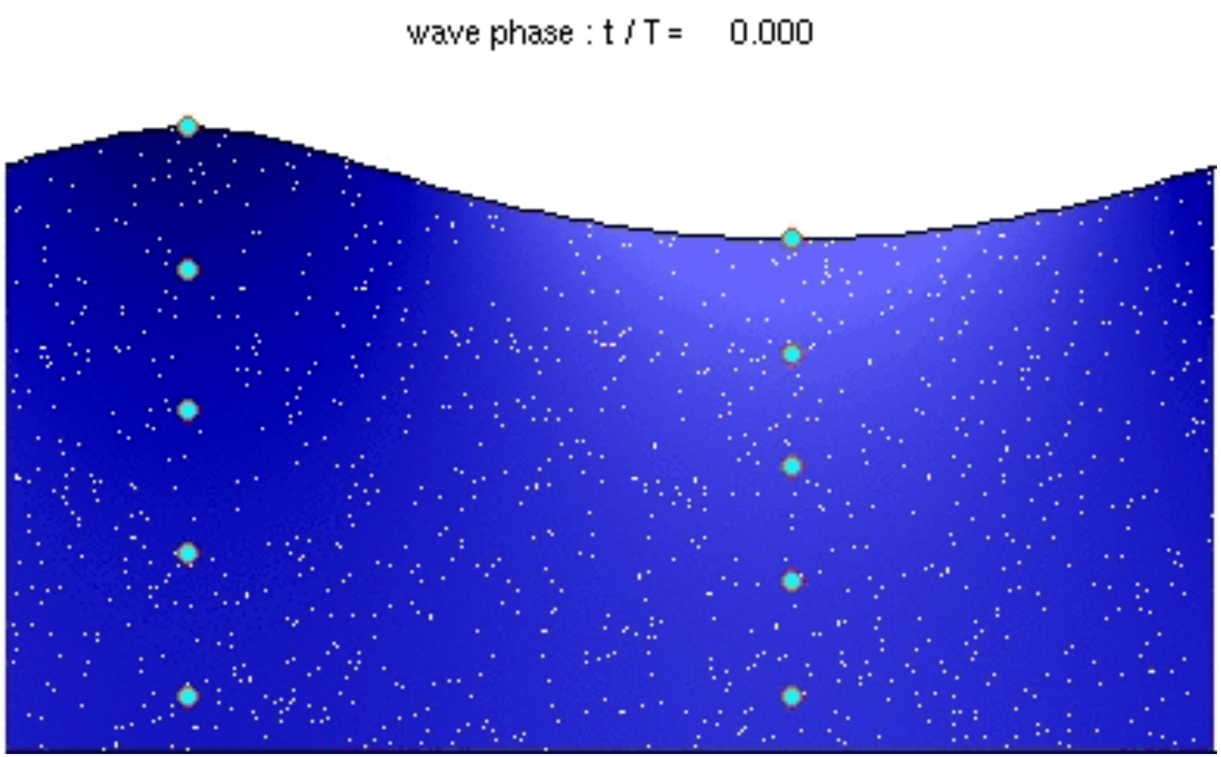


A lake in Chilean Patagonia

Wave-Induced Currents

$$U^S = \frac{1}{T}\int_0^T \left[(\text{displacements}) \bullet \nabla\right](\text{wave velocity})\,dt$$

- Stokes drift is the wave-averaged velocity following a particle.
- Lagrangian, not Eulerian.
- Decays steeply with depth.



The Wave-Averaged Boussinesq Equations

$$\underbrace{\partial_t \mathbf{u}}_{\text{Acceleration}} + \underbrace{(\mathbf{u}^L \cdot \nabla)\mathbf{u}}_{\text{Lagrangian Advection}} + \underbrace{f\hat{\mathbf{k}} \times \mathbf{u}^L}_{\text{Coriolis and Stokes Coriolis}} + \frac{\nabla p}{\underbrace{\rho_0}_{\text{Pressure}}} + \underbrace{u^{L,j}\nabla U^{S,j}}_{\text{Stokes Shear}} = \underbrace{b\hat{\mathbf{k}}}_{\text{Buoyancy}} + \underbrace{\nu\nabla^2\mathbf{u}}_{\text{Dissipation}}$$

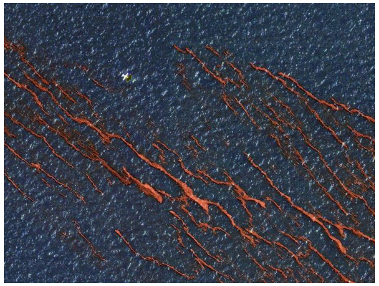
$$\partial_t \mathbf{u} + \left(\mathbf{u}^L \cdot \nabla\right) b = 0$$

$$\nabla \cdot \mathbf{u} = 0$$

Where, $\mathbf{u}^L = \mathbf{u} + \mathbf{U}^S$, and $\mathbf{U}^S(z)$ is prescribed

The Stokes Shear Force

* The Stokes shear force knocks the vertical momentum out of hydrostatic equilibrium.
* Horizontal variations in the Eulerian velocity induce LC.



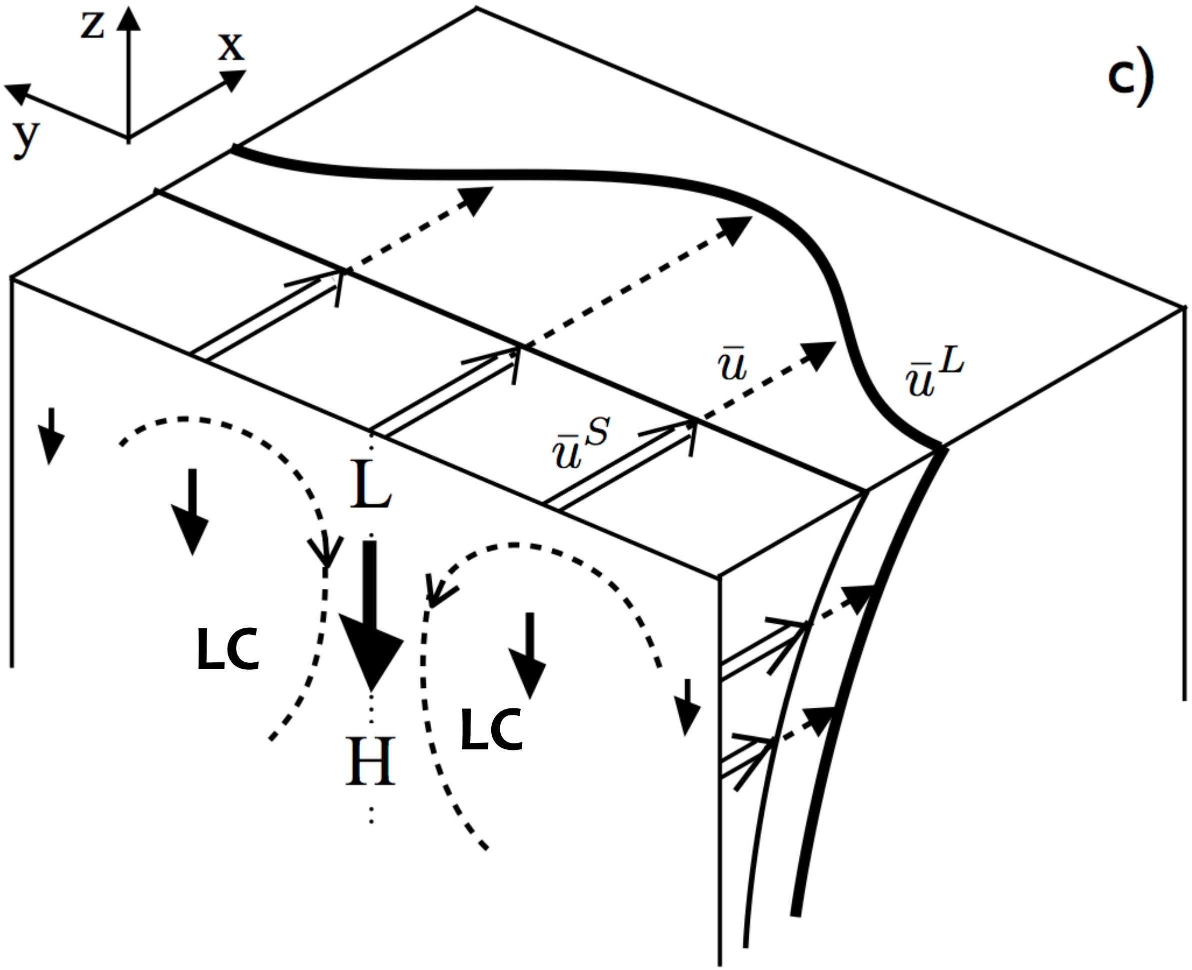


Figure 1c from Suzuki and Fox-Kemper 2015

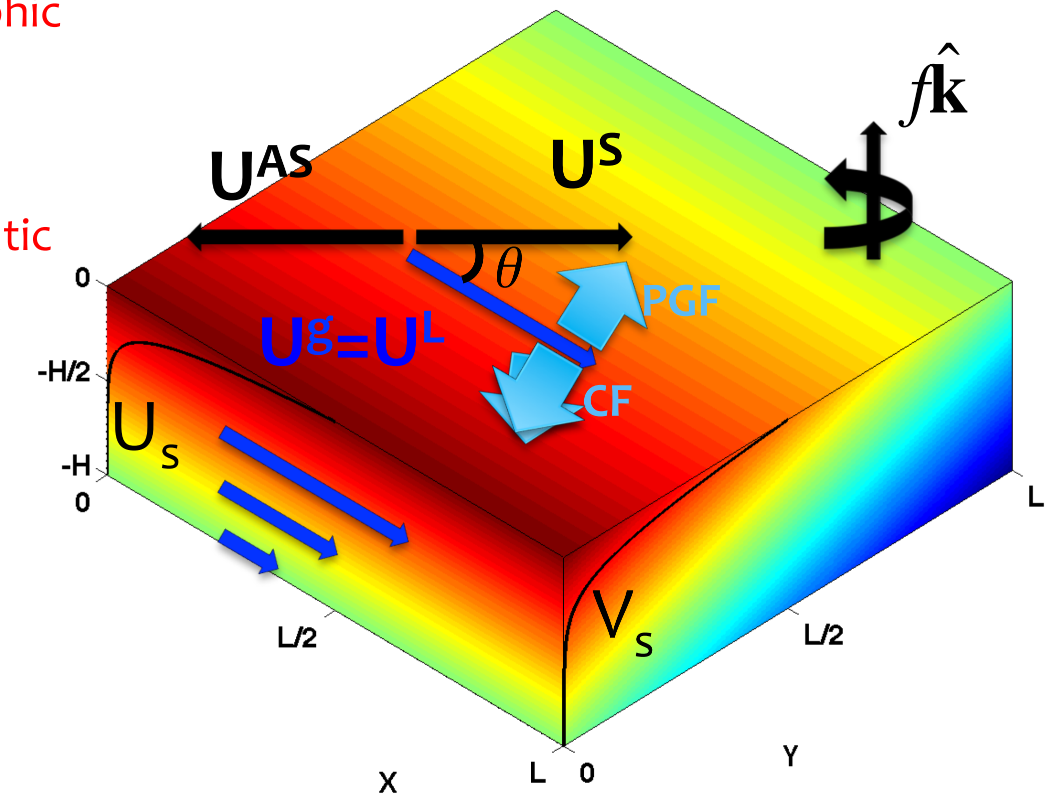
Lagrangian Thermal Wind Balance and Anti-Stokes Flow

Background Flow

$$f(\hat{\mathbf{k}}\times\bar{\mathbf{U}}^L) = -\frac{\nabla\bar{P}}{\rho_0} \quad \Big\} \text{ Geostrophic}$$

$$\bar{P}_z = \bar{B} \quad \Big\} \text{ Hydrostatic}$$

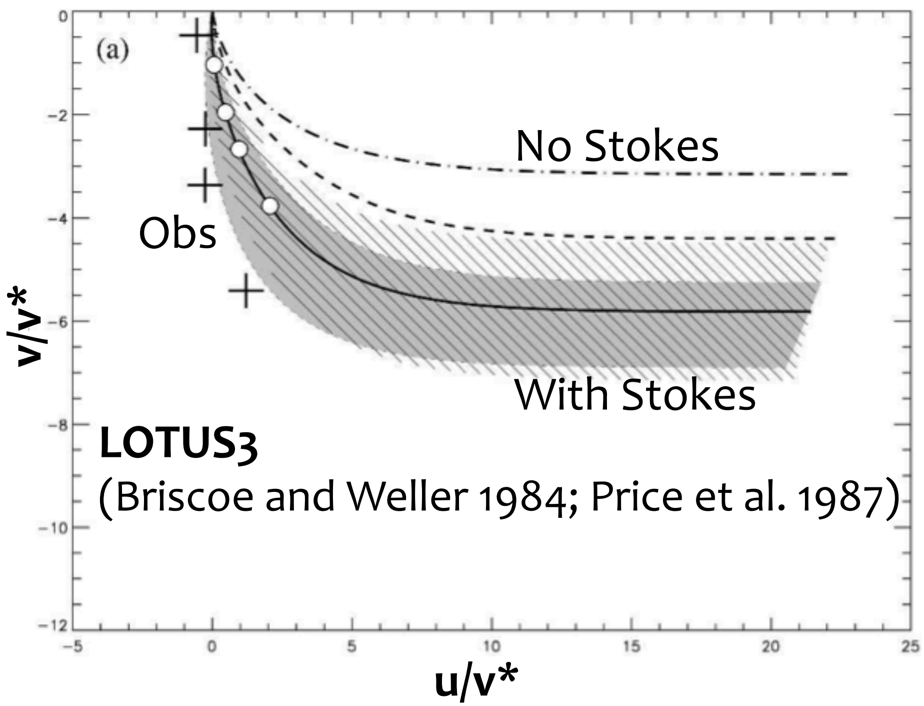
$$f\bar{\mathbf{U}}_z^L = -\bar{B}_Y \quad \Big\} \text{ Thermal Wind}$$

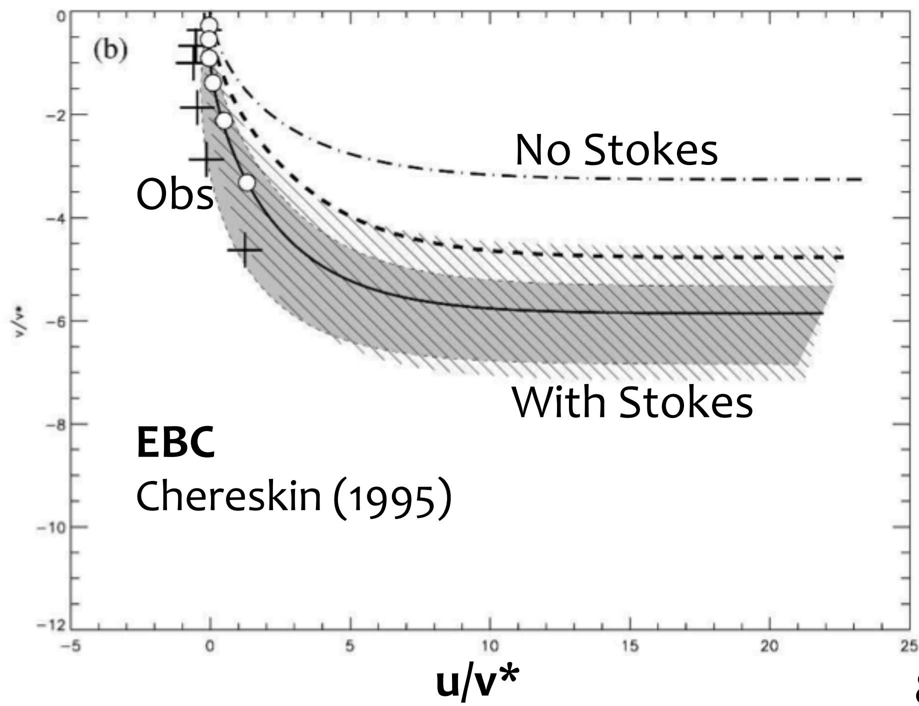


Observational Evidence of “Anti-Stokes Flow”

* Accounting for Stokes drift in the Ekman spiral may account for differences between analytic solutions and observations

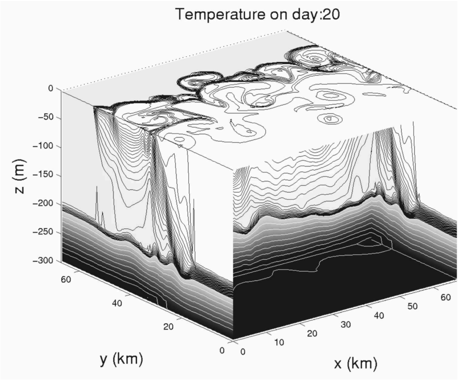
Polton et al. 2005





Geostrophic Instabilities

* Switching A and C is stable.
* Switching A and B is unstable.
* Once the flow gets moving, the Coriolis force turns it to the right



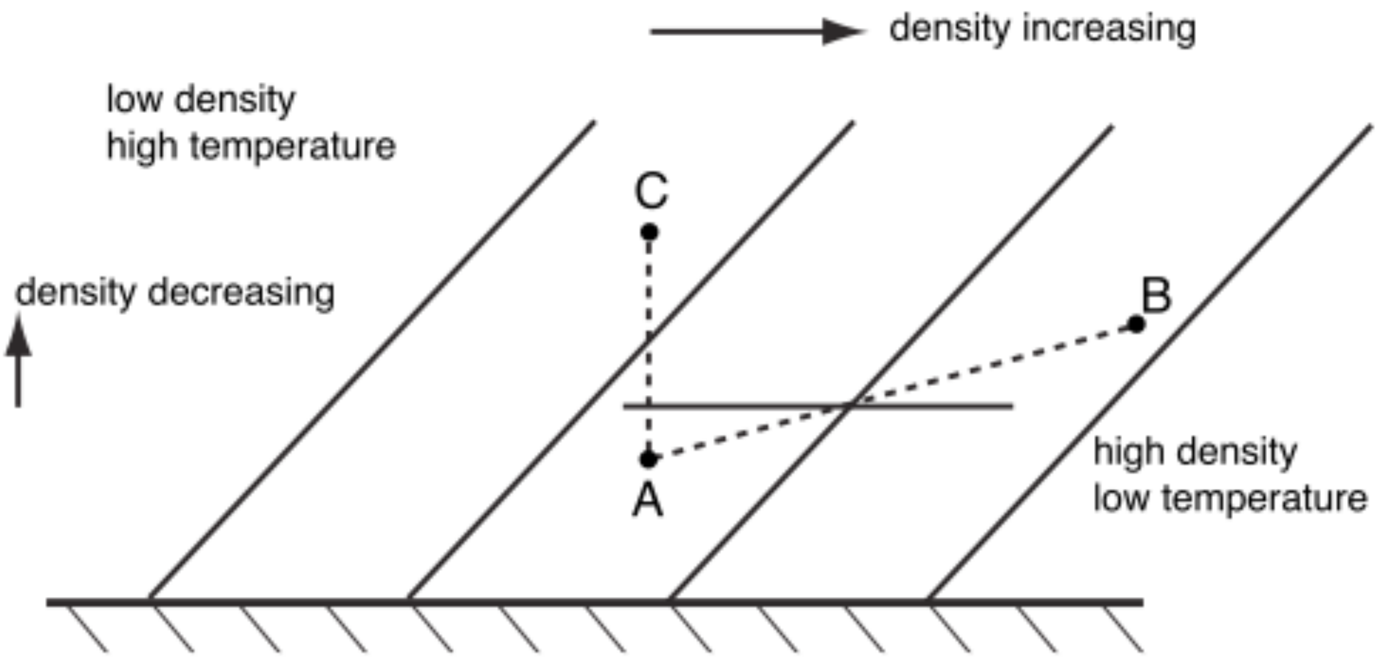


Figure 6.9 from Vallis, 2006

Analytic Stability Criteria: Geostrophic Instabilities

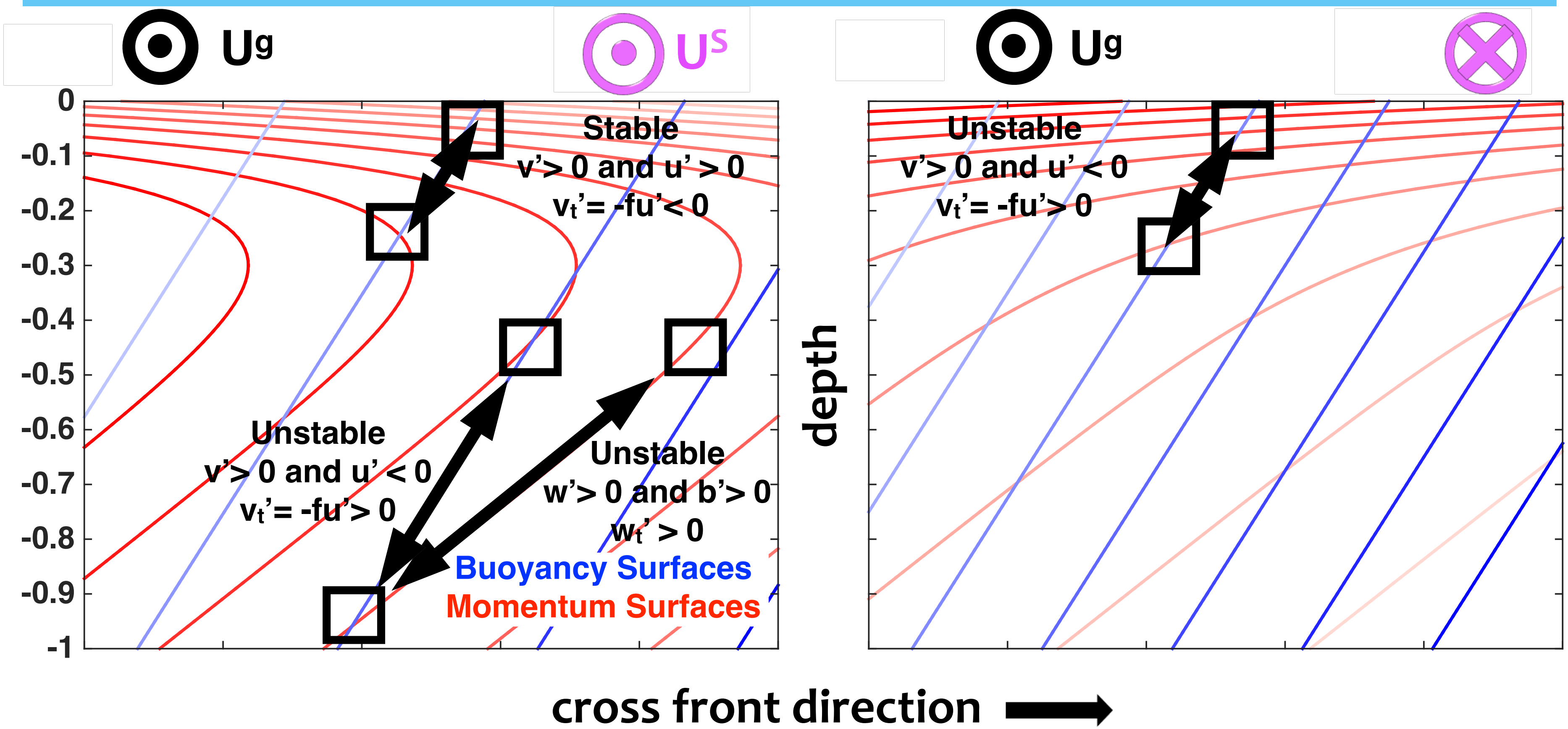
* Charney, Stern, and Pedlosky showed, that geostrophic instability exists only if any of the following is true:

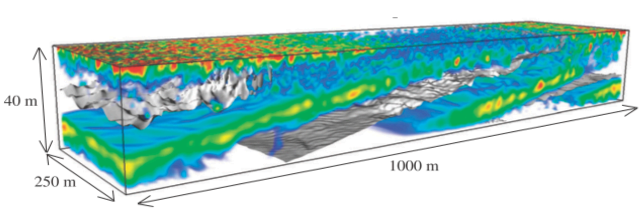
1. Q_y changes sign in the interior of the domain.
2. Q_y is the opposite sign to $U^L{}_z$ at the surface.
3. Q_y is the same sign to $U^L{}_z$ at the bottom.
4. $U^L{}_z$ has the same sign at the surface and bottom.

Where Q is the quasi-geostrophic potential vorticity:

$$\bar{Q} = \nabla_H^2 \bar{\psi} + \beta Y + \partial_z \left(\frac{f_0^2}{N^2} \bar{\psi}_z^L \right)$$

Symmetric Instabilities





Analytic Stability Criterion: Symmetric Instability

- The Ertel potential vorticity (PV) depends on the alignment of the vorticity an buoyancy gradient.

$$\bar{Q} = \left(\nabla \times \bar{\mathbf{U}} + f\hat{\mathbf{k}}\right) \bullet \nabla \bar{B}$$

- The Stokes drift (Lagrangian mean of the leading order, ***irrotational*** wave velocity) produces no vorticity. However, if the flow is in Lagrangian thermal wind balance, the vorticity is modified by the anti-Stokes Eulerian flow.
- Hoskins (1974) showed that if a front in thermal wind balance is symmetrically unstable, the PV must be negative.
- Extends to flows in Lagrangian thermal wind balance in the special case:

$$U^S = \mu z, \quad \mathrm{V}^S = 0$$

$$SI \;\Rightarrow\; f\bar{Q} = \underbrace{f^2 N^2 - M^4}_{\text{Geostrophic}} - \underbrace{f M^2 U_z^S}_{\text{Anti-Stokes}} < 0$$

Linear Stability Method

* Rescale equation
* Multiple scales of horizontal variation: $x, X, y, Y, t, T.$
* Decompose into mean and perturbation:

$$\mathbf{u} = \bar{\mathbf{U}}(X,Y,z,T) + \mathbf{u}'(x,X,y,Y,z,t,T)$$

* Find a solution to the mean (averaged over x,y) equations
* Force the perturbation equations with the mean flow solution
* Assume

$$u' = \tilde{u}(z)\, e^{i(kx+ly+\sigma t)}$$

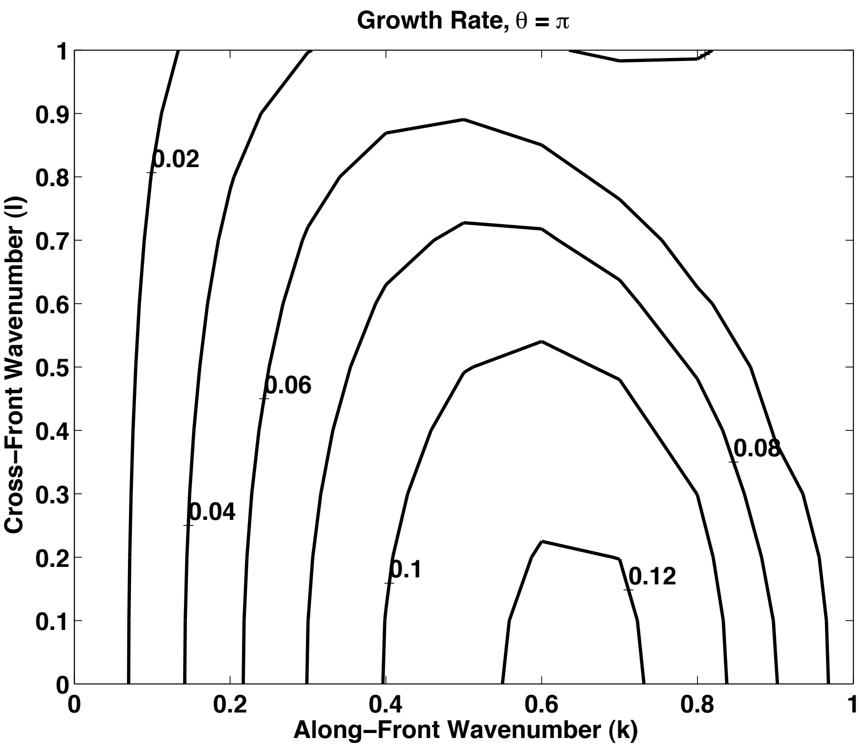
Vertical structure of instabilities   Growth rate of instabilities

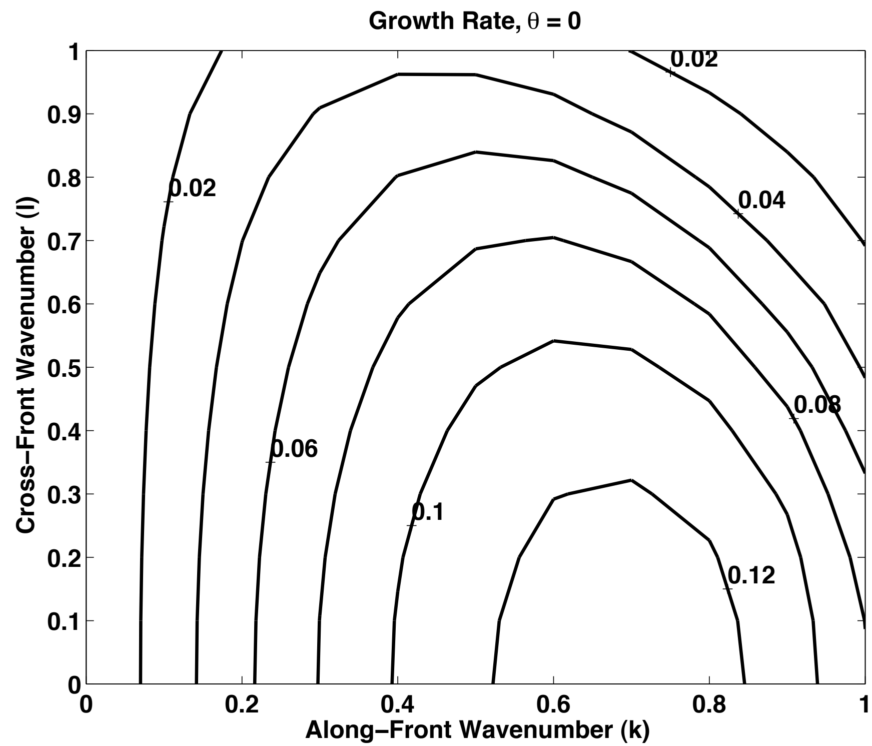
$$\tilde{u}, \sigma = F\left(Ri, \mu\lambda, \gamma, \lambda, \theta, Ek, Ro, \alpha\right)$$

$$Ri = \frac{N^2}{U_z^{L,2}} = \frac{\text{Vertical Stratification}}{\text{Lagrangian Shear Squared}} \qquad \mu\lambda = \frac{U_z^S}{U_z^L} = \frac{\text{Stokes Shear}}{\text{Lagrangian Shear}}$$

Geostrophic Instabilities

* When the Stokes drift and geostrophic flow are aligned, the anti-Stokes flow yields reduced Eulerian shear.
* Less Eulerian shear near the surface results in higher growth rates and wavenumbers for GI.

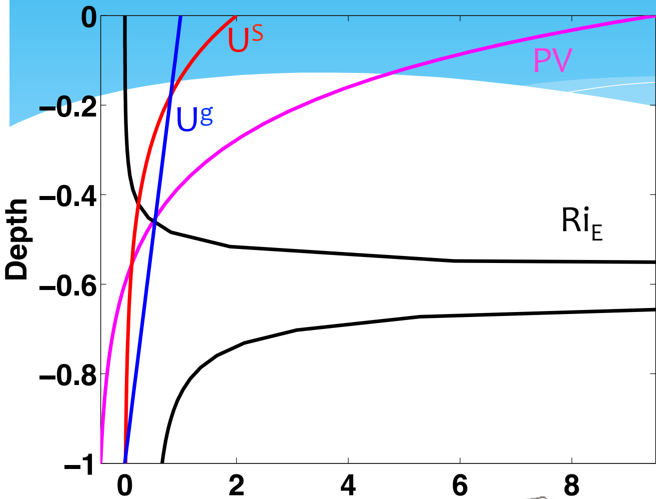




~~Ri <1 ⇒ SI~~

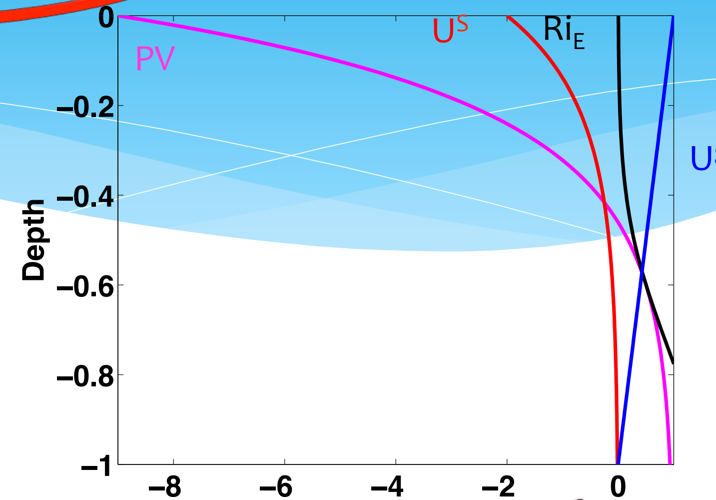
Ri = 0.5

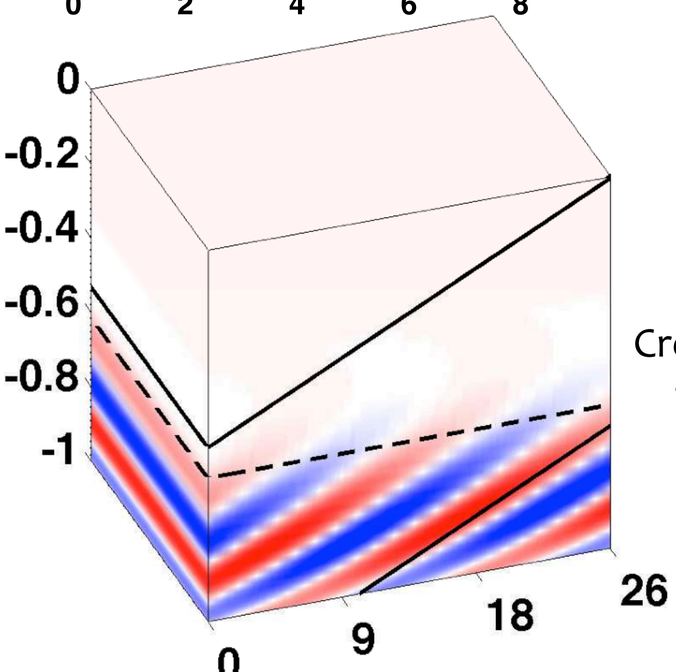
Background State



Ri = 2

Background State

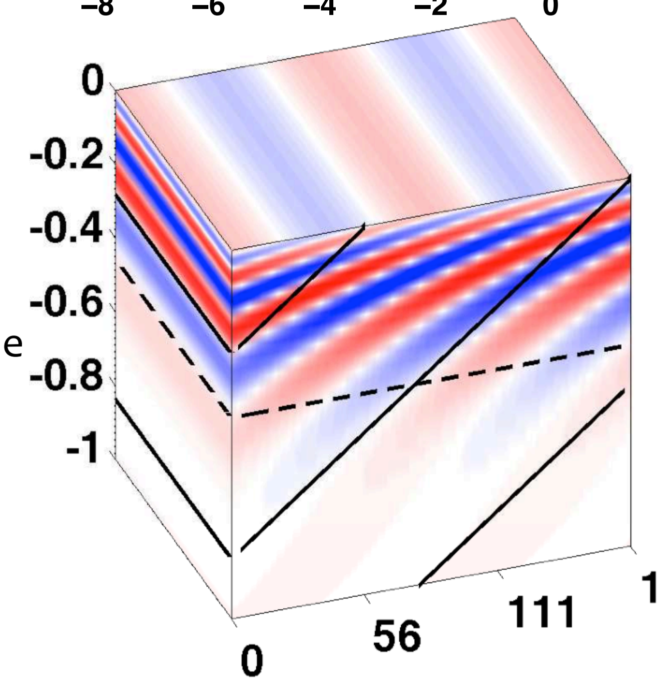




—— Isopycnals

- - PV=0

Cross front velocity for the fastest growing mode



Energetics

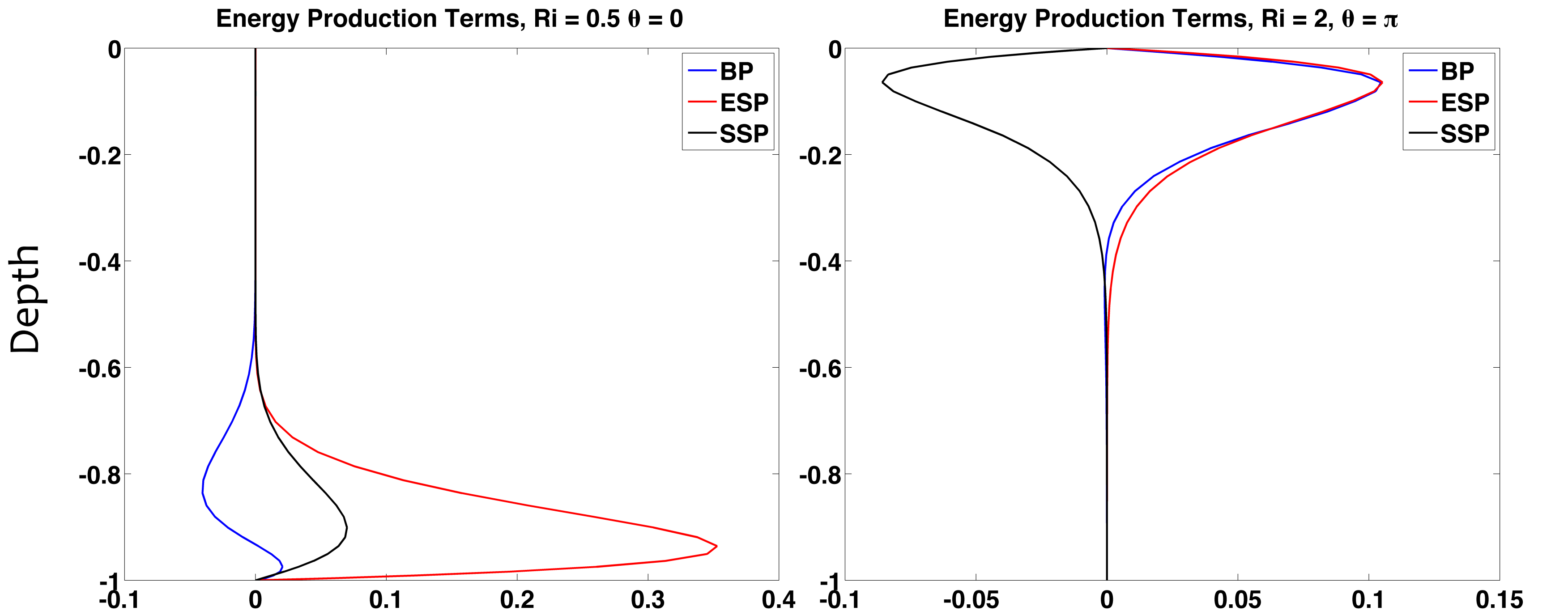
* Energetics are a useful tool to distinguish modes.

$$\frac{\overline{D^L e'}}{Dt} = \underbrace{-\overline{\mathbf{u}'w'} \cdot \overline{\mathbf{U}}_z}_{\text{ESP}} \underbrace{-\overline{\mathbf{u}'w'} \cdot \mathbf{U}_z^S}_{\text{SSP}} \underbrace{-\overline{w'b'}}_{\text{BP}} - PW + D$$

* BP dominant: instability extracts potential energy to RE-stratify the mixed layer (typical of GI).
* SSP, ESP dominant: instability extracts kinetic energy (typical of SI, LC, KH)
* Hybrid modes with various mixed of energy production terms exist.

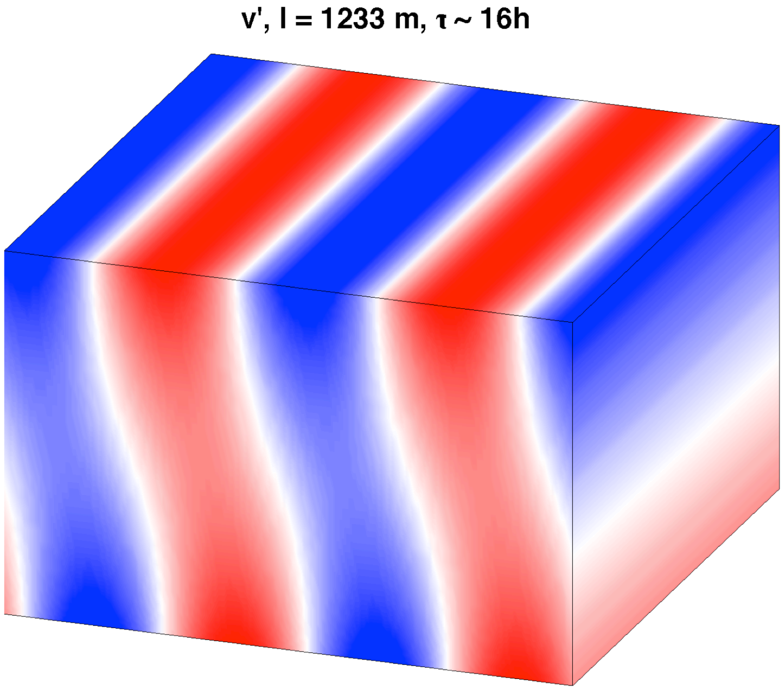
Stokes Drift Induces more Restratification by SI

- Stokes drift changes the path along which SI move, favoring more cross isopycnal motion near the surface.
- This increases BP (restratification).
- Anti-aligned Stokes drift $\Rightarrow$SSP<0 (the work done by the Stokes shear force).

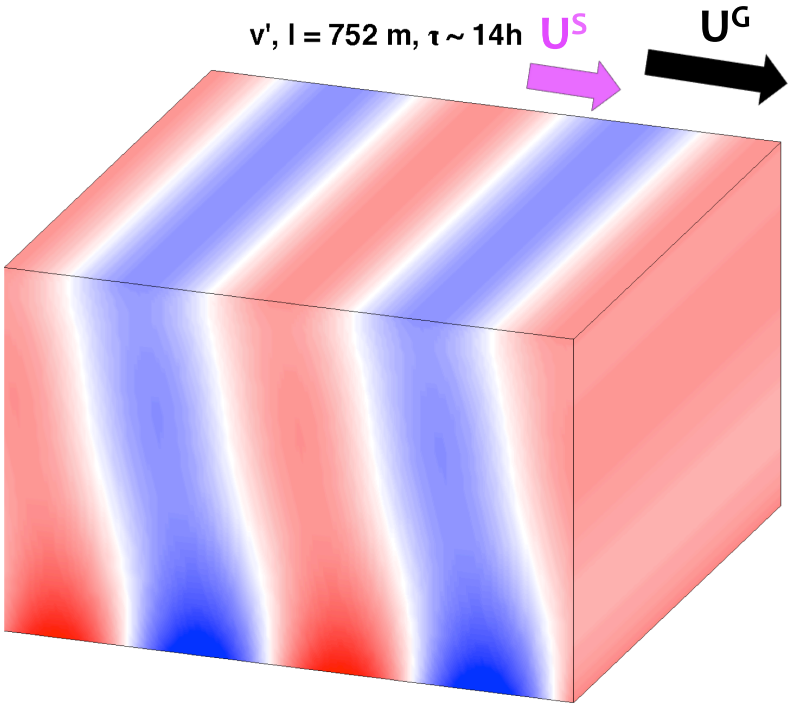


Stokes-Modified GI

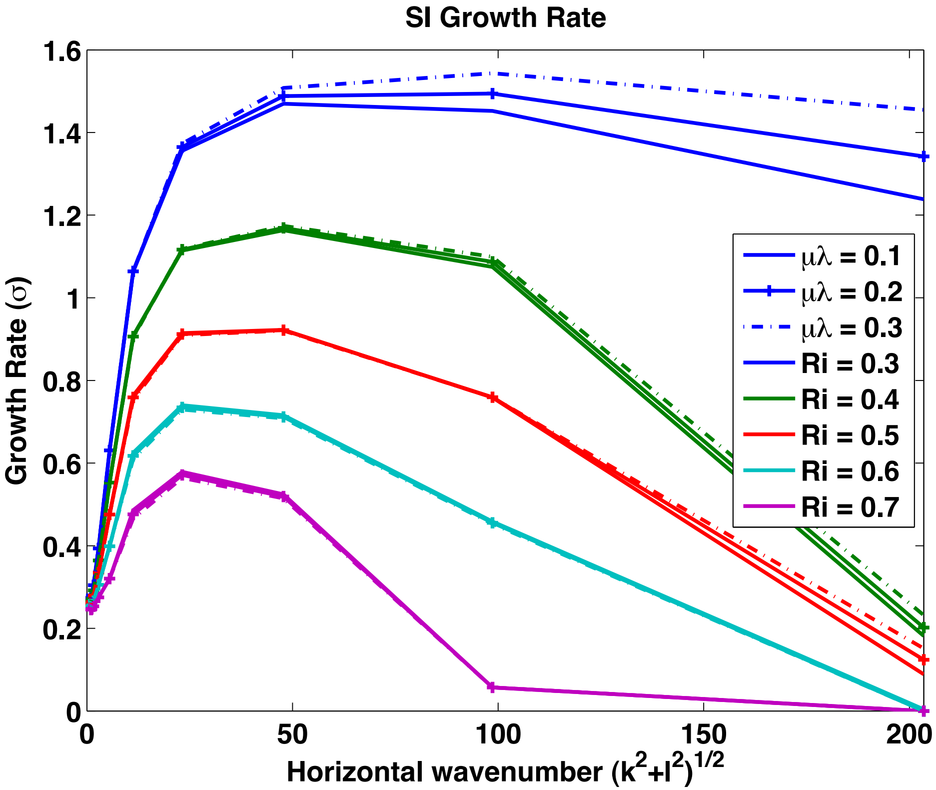
No Stokes

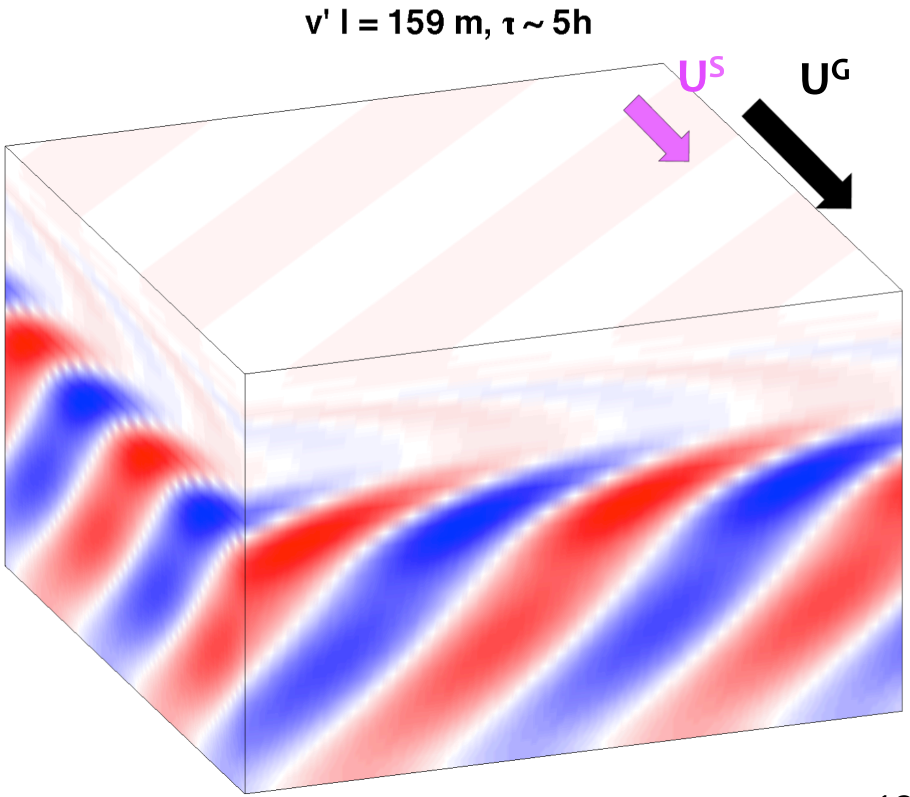


with Stokes

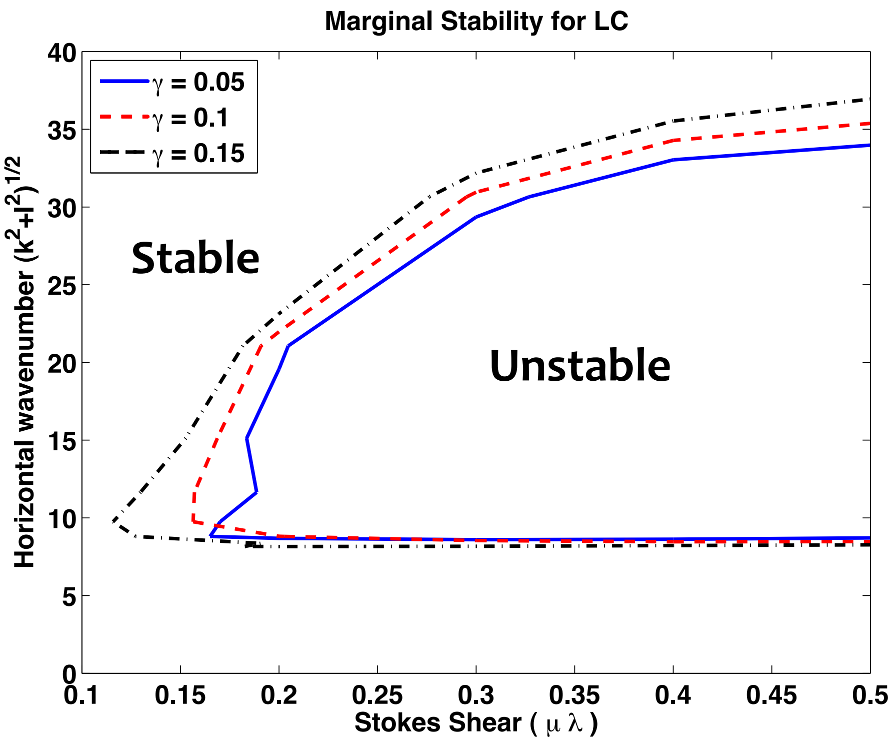


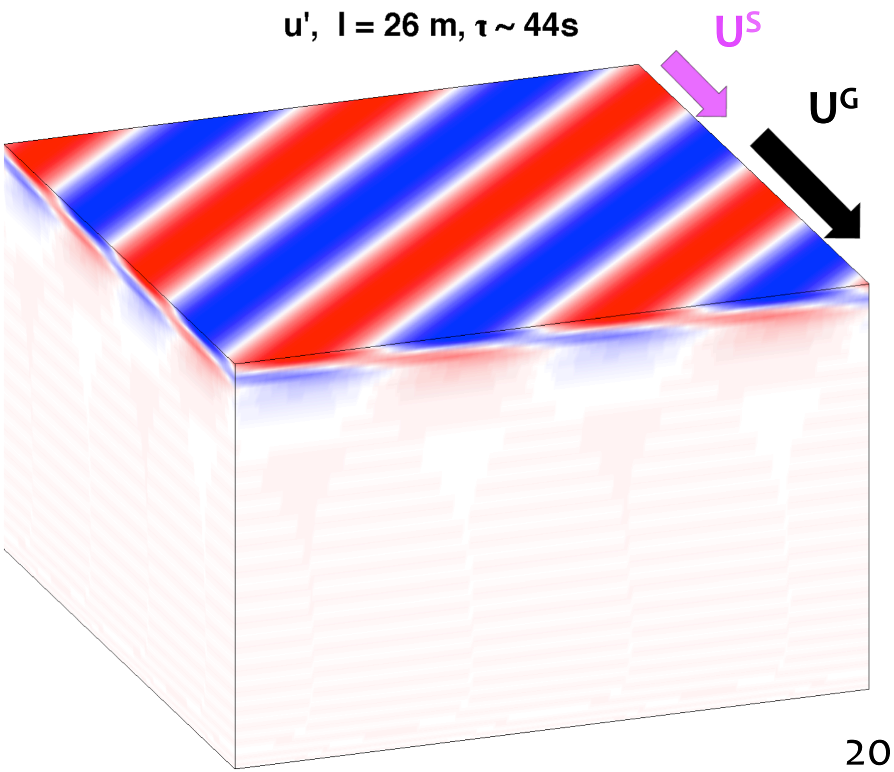
Stokes-Modified SI





LC within a Front





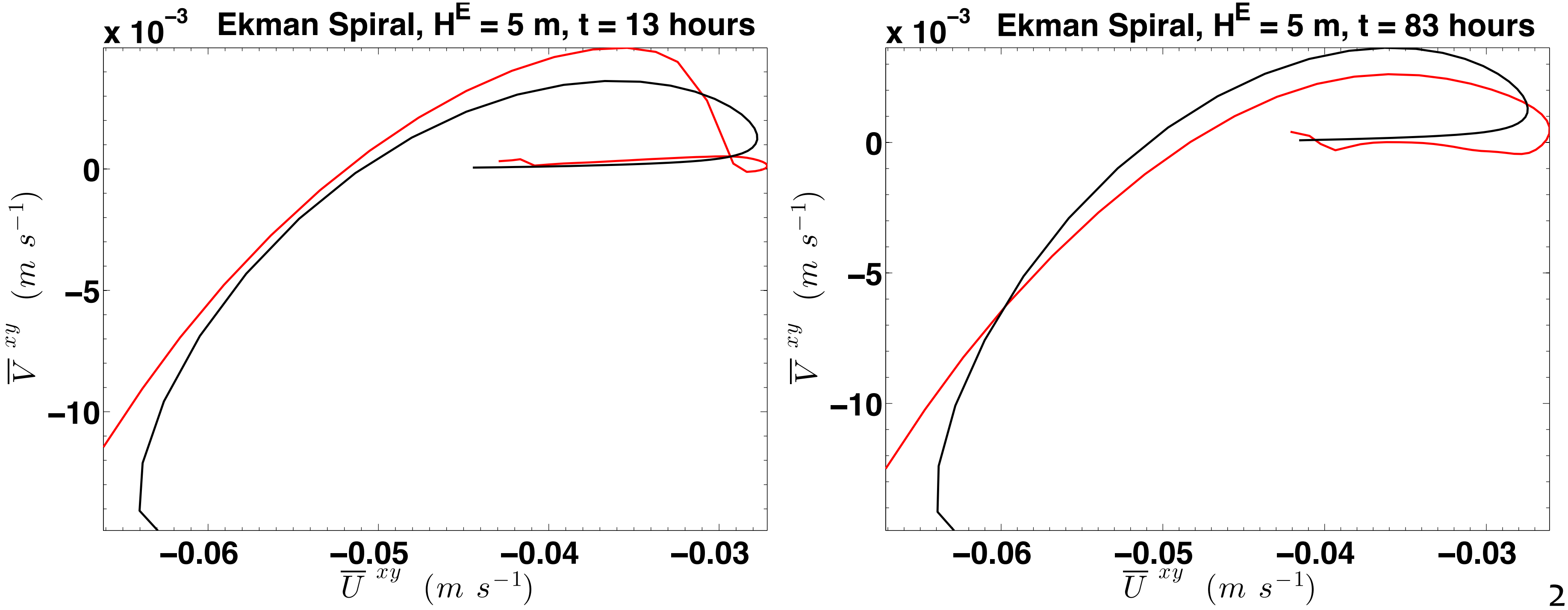
Nonlinear Simulations

* National Center for Atmospheric Research (NCAR) LES model.
* Configuration
  * 500m (along-front) x 8km (cross-front) x 75m deep
    * Intentionally along-front limited to prohibit GI. Simulating GI and LC would require O(10^6 cpuh) vs the O(10^4 cpuh) required for the simulations performed.
  * ~4m horizontal x ~1m vertical resolution.
  * Periodic BC's in the horizontal (requires simulating 2 fronts)
  * No flux on top and bottom
  * No wind stress on top
* Cases
  1. PV<0, no Stokes (Ri = 0.5, μ = 0): control case
  2. PV<0 at depth, with Stokes (Ri = 0.5, μ = 2)
  3. PV>0 at depth, with Stokes (Ri = 2, μ = 1)

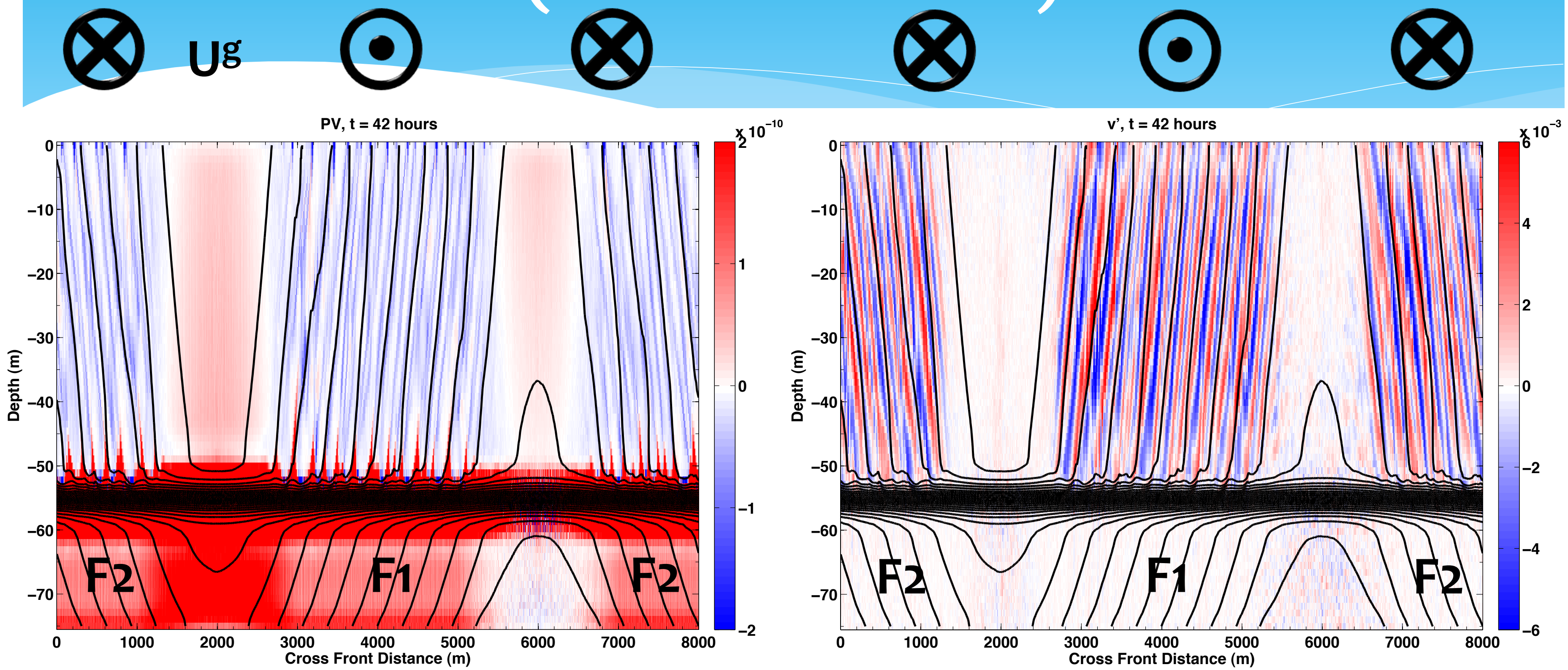
Stokes-Ekman-Front Layer

– Analytic Solution

– Horizontal average from LES



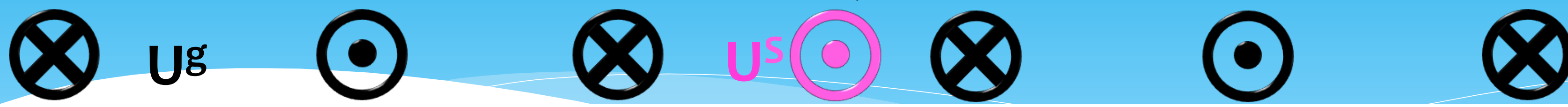
LES Case 1 (Control Case)

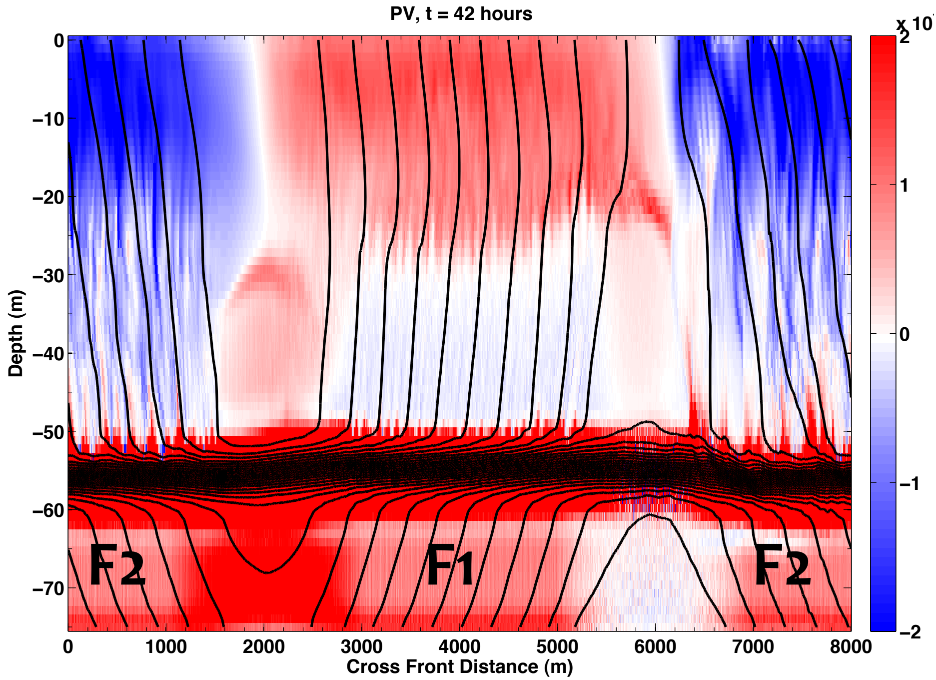


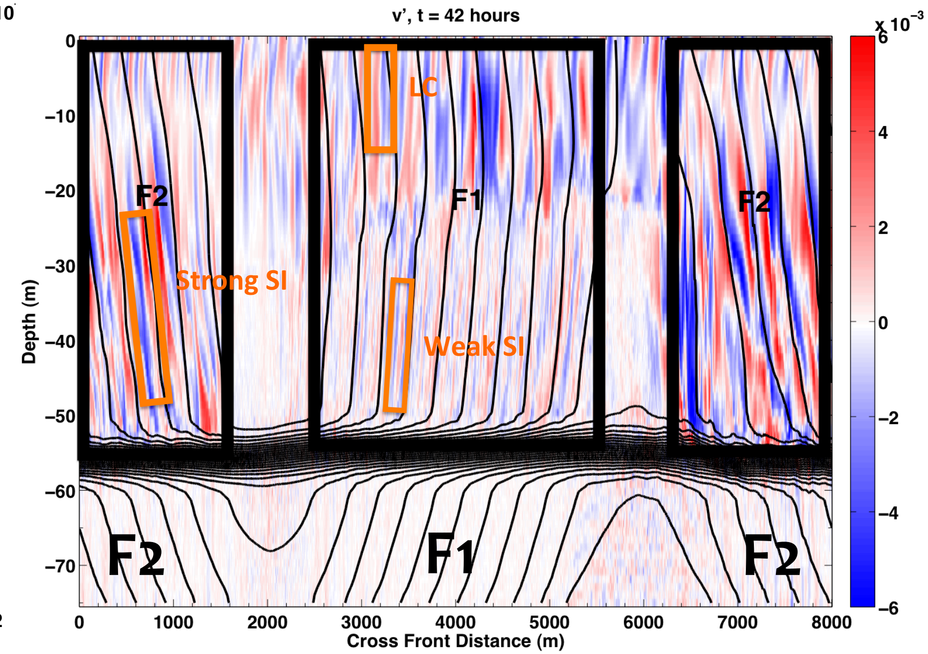
- PV<0, no Sokes (Ri = 0.5, μ = 0)
- We expect SI in the within the fronts

LES Case 2

Ri = 0.5, $\mu = 2$



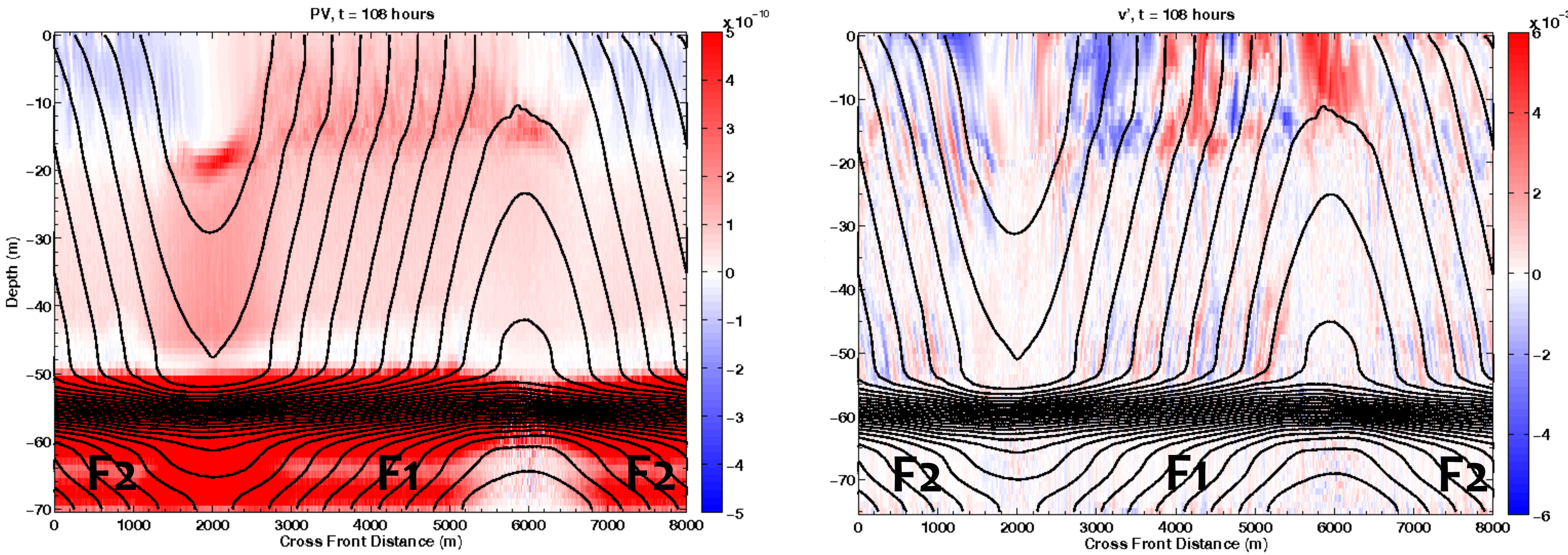




* We expect SI in the regions where PV<0
* Stokes-Ekman-Front layer yields an Ekman transport to the left, destabilizing F1 while stabilizing F2.

LES Case 3

Ri = 2, μ = 1

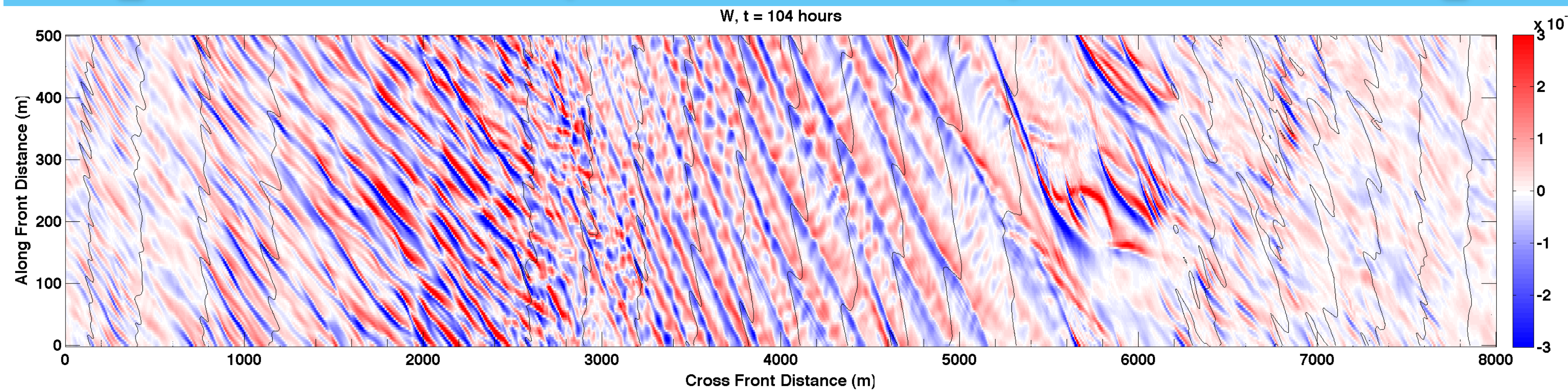


* We expect SI in the regions where PV<0
* Stokes-Ekman-Front layer yields an Ekman transport to the left, destabilizing F1 while stabilizing F2.

Vertical Stratification Slows LC

Horizontal slice of vertical velocity at ~ 5m deep.

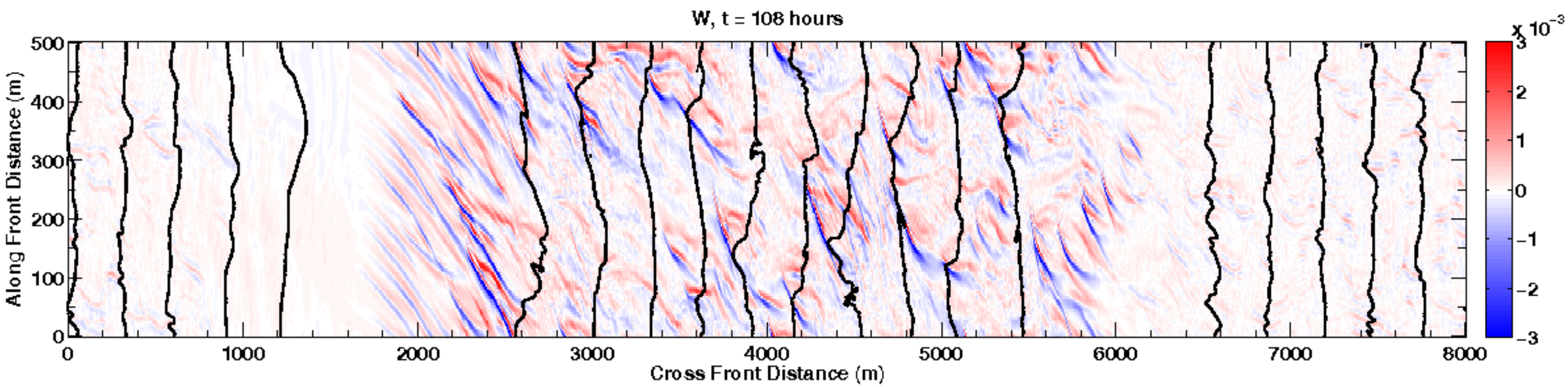
U^S Ri = 0.5, μ = 2 U^G



HOT, LIGHT

Ri = 2, μ = 1

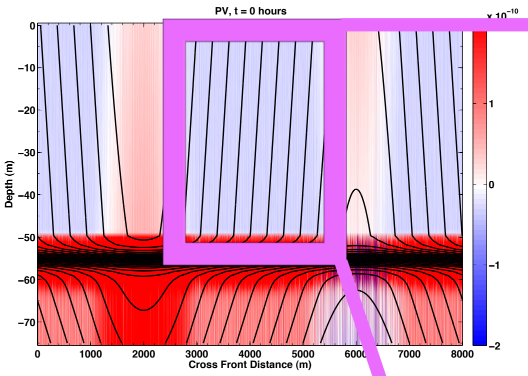
COLD, DENSE

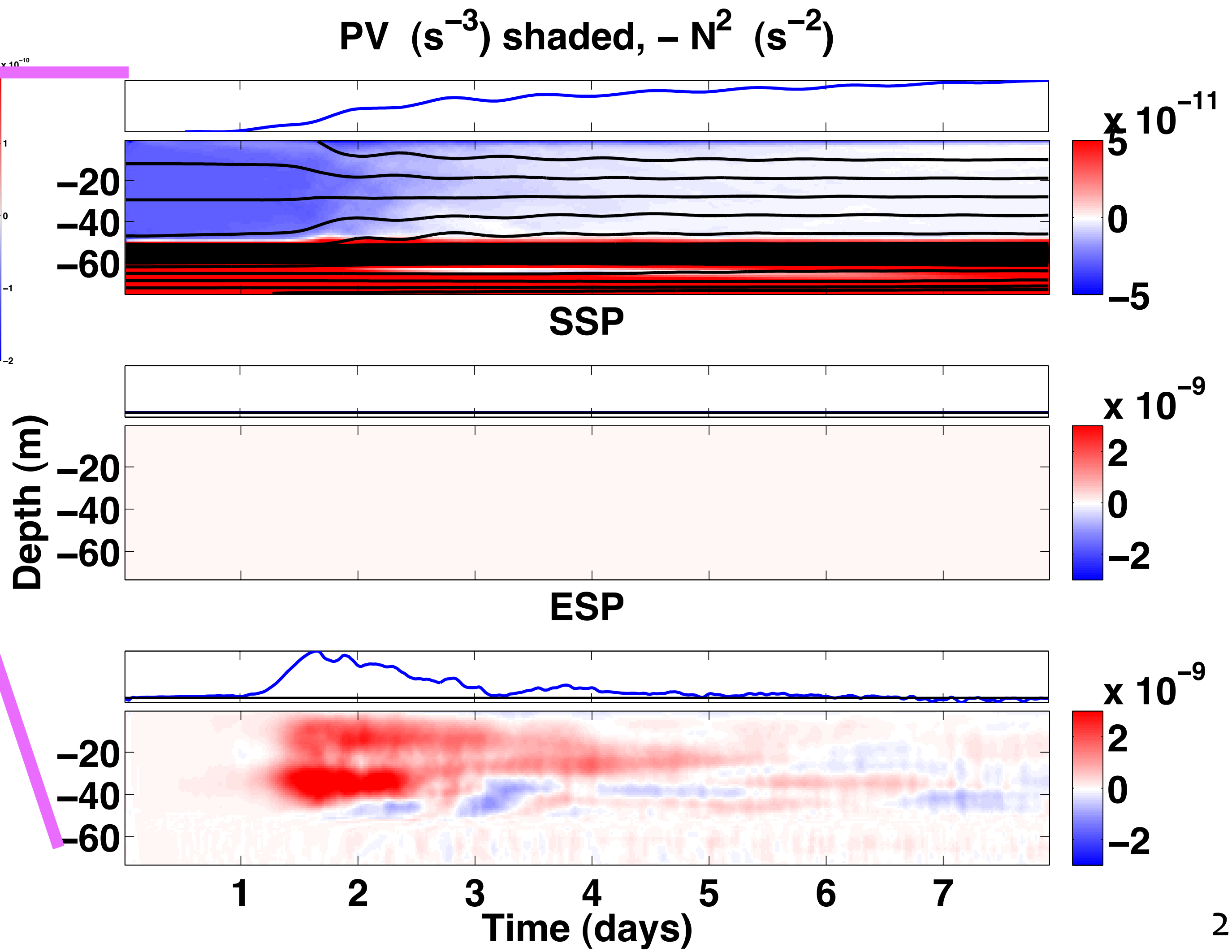


Energetics and PV: Control Case

Ri = 0.5, μ = 0

F1

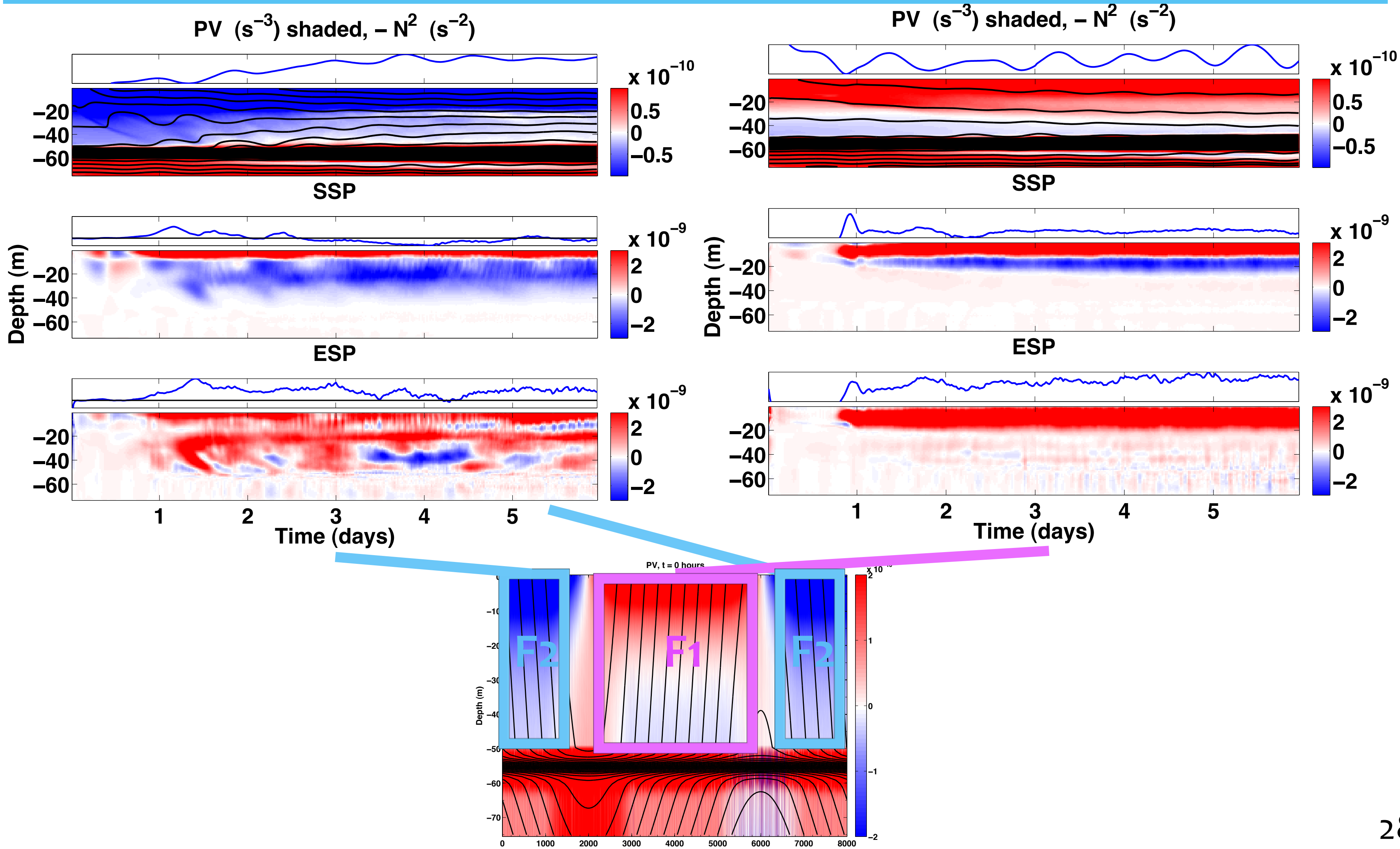




* Shading: horizontal averages within the front.
* Line plots: volume averages within the front.

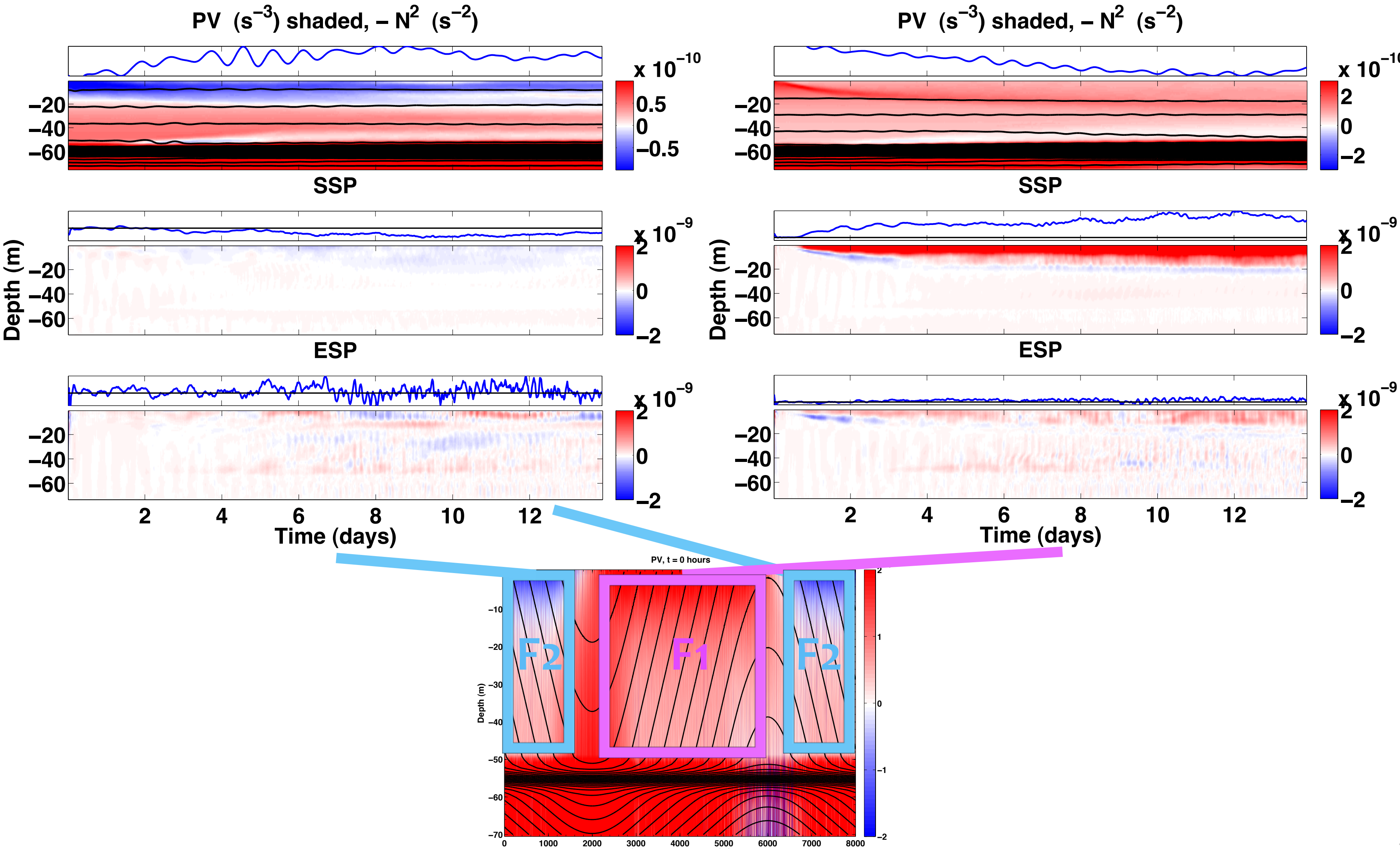
Energetics and PV: Case 2

Ri = 0.5, μ = 2



Energetics and PV: Case 3

Ri = 2, μ = 1



Conclusions

* GI are only weakly affected by Stokes drift, but their instability depends on the Lagrangian, not just Eulerian shear.
* If the flow is indeed unstable to GI, increased anti-Stokes Eulerian shear reduces the growth rate and wavenumber.
* If the flow is unstable to SI, then PV<0 (and the implication appears to go the other way as well), and anti-Stokes Eulerian flow modifies the PV.
  * Observational estimates of PV must be based on Eulerian shear if SI are of interest.
* Stokes forced SI do more BP that their no Stokes counterparts.
* SSP does work against SI when the Eulerian and Stokes shears oppose each other.
* Stokes drift can indirectly induce restratification (rather than mixing with LC) by modifying the PV and shear, causing SI to do more BP.
* LC are suppressed by the Ekman induced restratification of the front.

Thanks!

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