Interactions Between Surface Gravity Wave Groups and Deep Stratification in the Ocean

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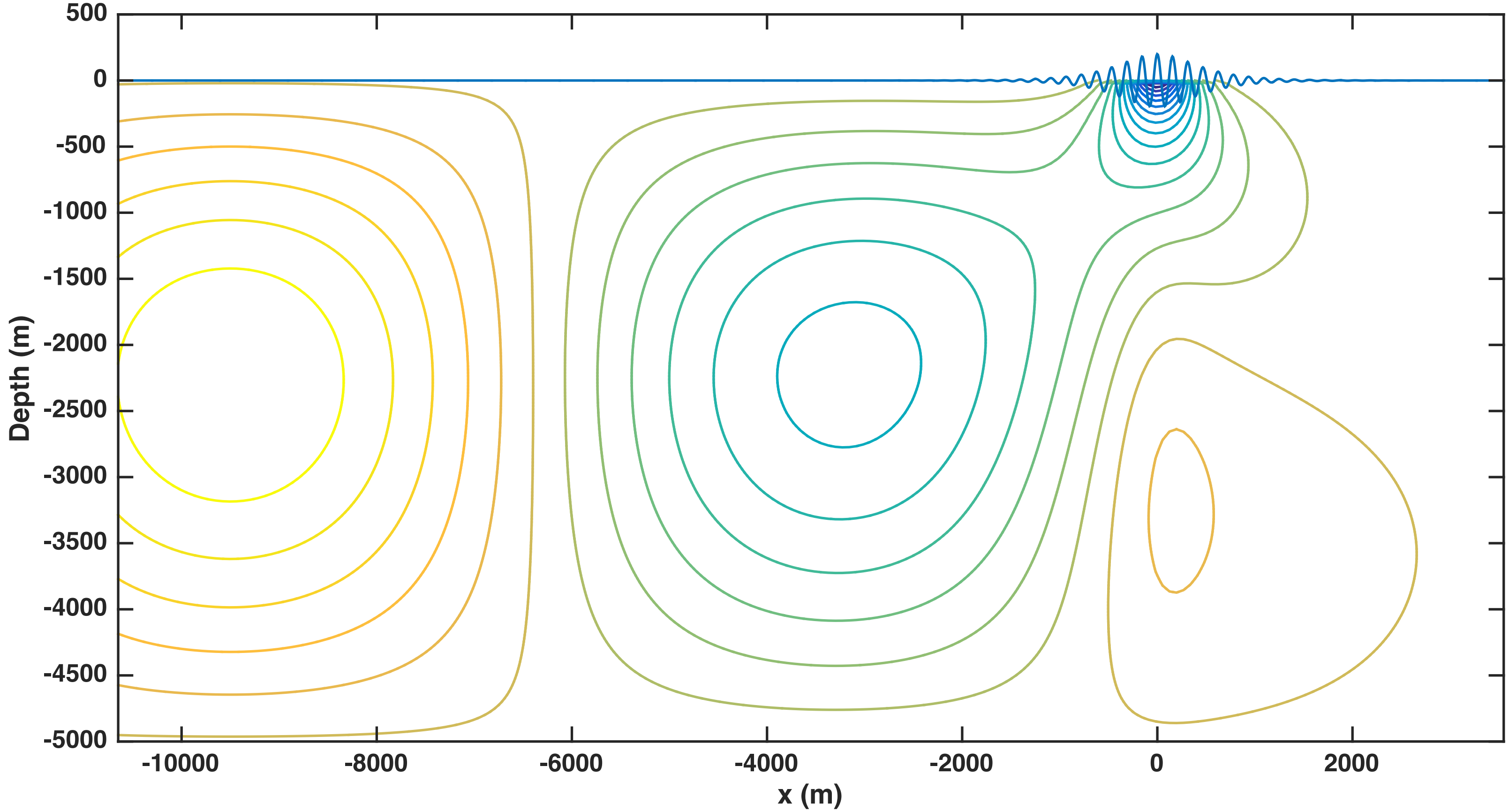
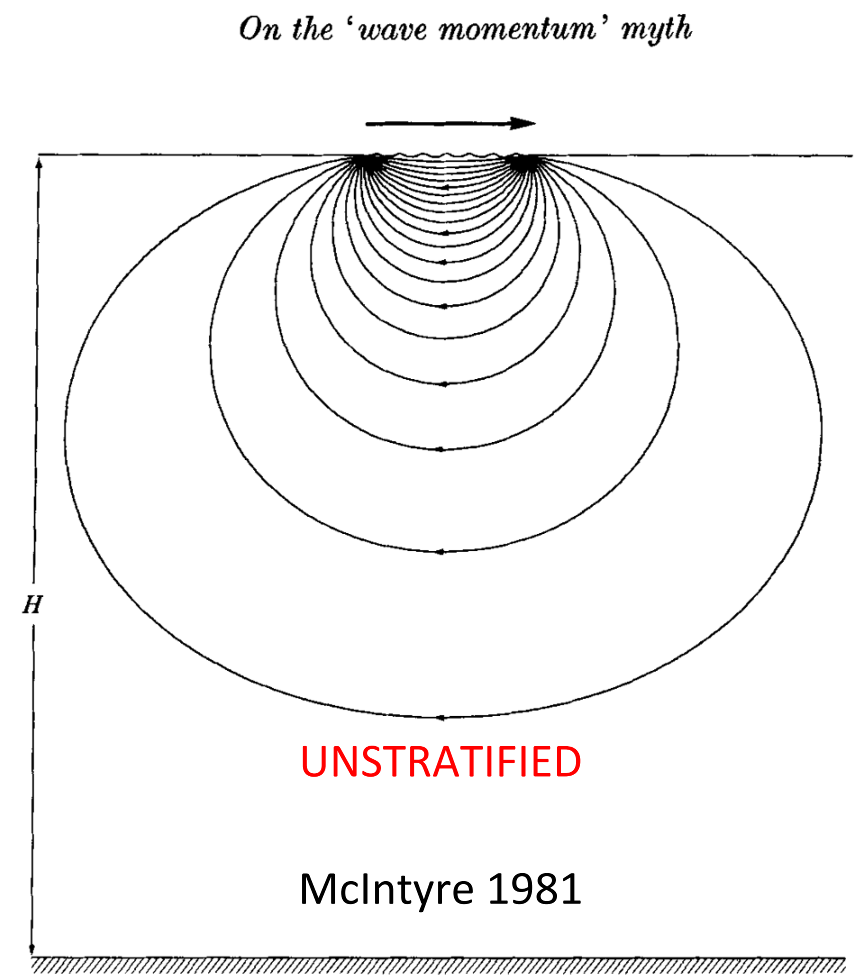


The Eulerian Return Flow

- Longuet-Higgins and Stewart (1964) showed that there is a deep (decaying algebraically with depth) Eulerian return flow beneath groups of surface waves.
- McIntyre (1981) comments: "…in real oceanographic applications the existence of stable stratification can greatly modify the form of the return flow, coupling it directly to internal gravity waves."



Overview and Scales

SW – Surface Waves

IW – Internal Waves

a_* – SW amplitude $\sim 2m$

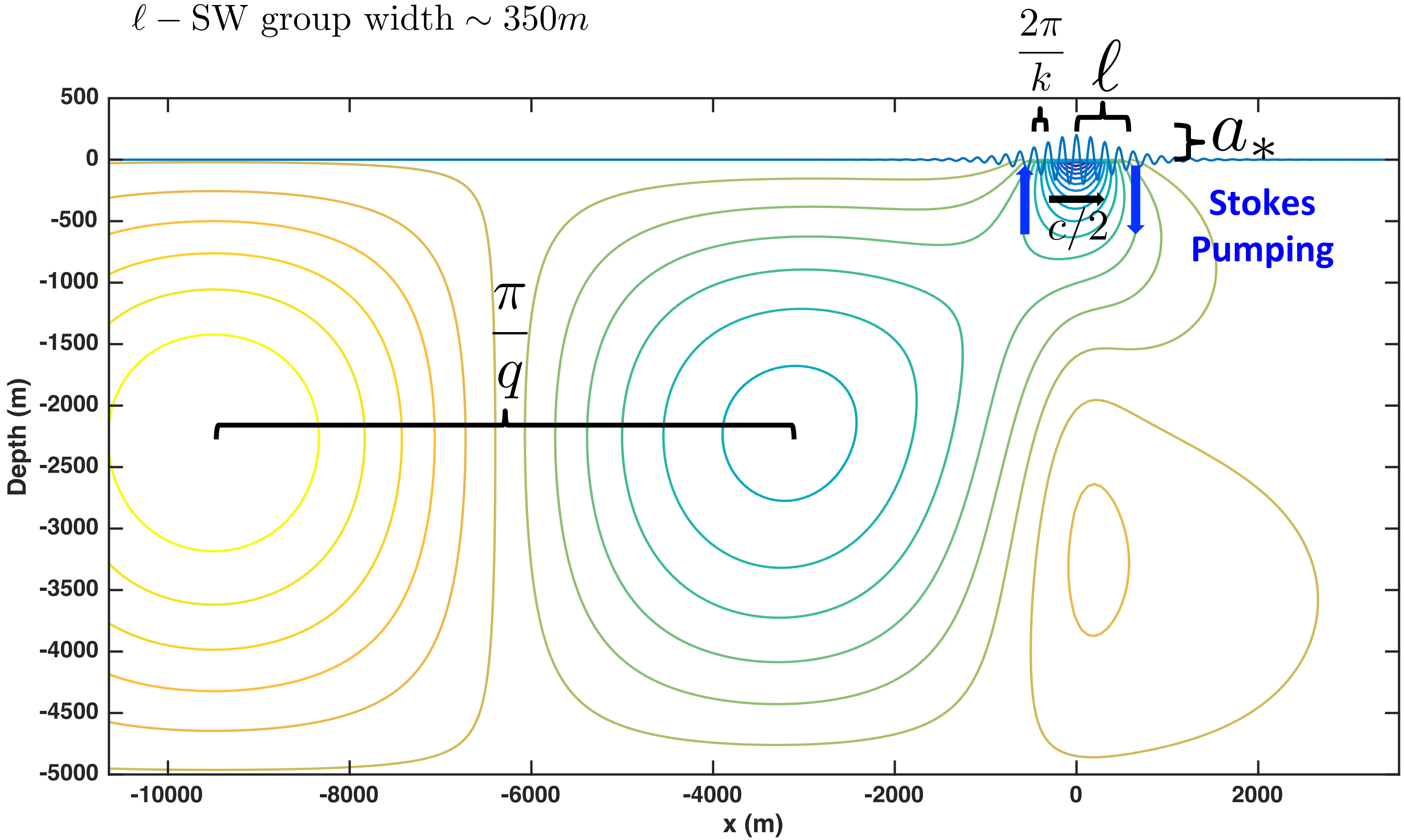
k – SW wavenumber $\sim 2\pi/(100m)$

ℓ – SW group width $\sim 350m$

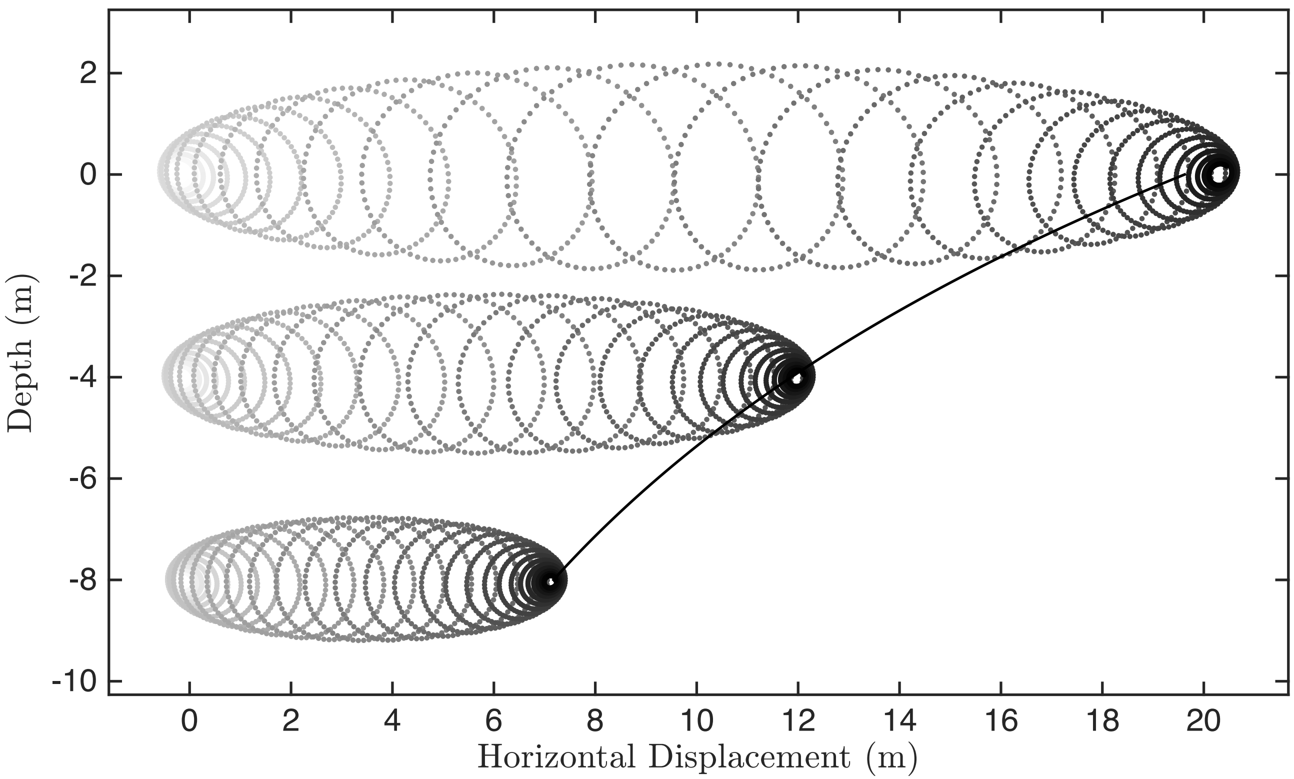
c – SW phase speed $\sim 12.5ms^{-1}$

q – IW wavenumber $\sim 2\pi/(6.7km)$

$$v \text{ – } \underbrace{\text{IW phase speed in 2D} = \frac{c}{2}}_{\text{2D resonance condition}} \sim 6.25ms^{-1}$$



Stokes Drift for Surface Wave Groups



$$\boldsymbol{u}^S = (a_* k)^2 \, c e^{-\left(\frac{x - \frac{c}{2} t}{\ell}\right)^2 + 2kz}$$

$$\begin{aligned} \boldsymbol{d}^S &= \int_{-\infty}^{\infty} \boldsymbol{u}^S dt \\ &= 2\sqrt{\pi}\ell \, (a_* k)^2 \, e^{2kz} \\ &\approx 8 \ m \end{aligned}$$

Surface Boundary Forcing

- The deep return flow is forced by the divergence of the Lagrangian mean mass flux.

Neglect surface displacement (set up/down) due to wave group

$$@z = 0, \quad \overline{w} = \cancel{\overline{\zeta}_t} + (\overline{u\zeta})_x$$

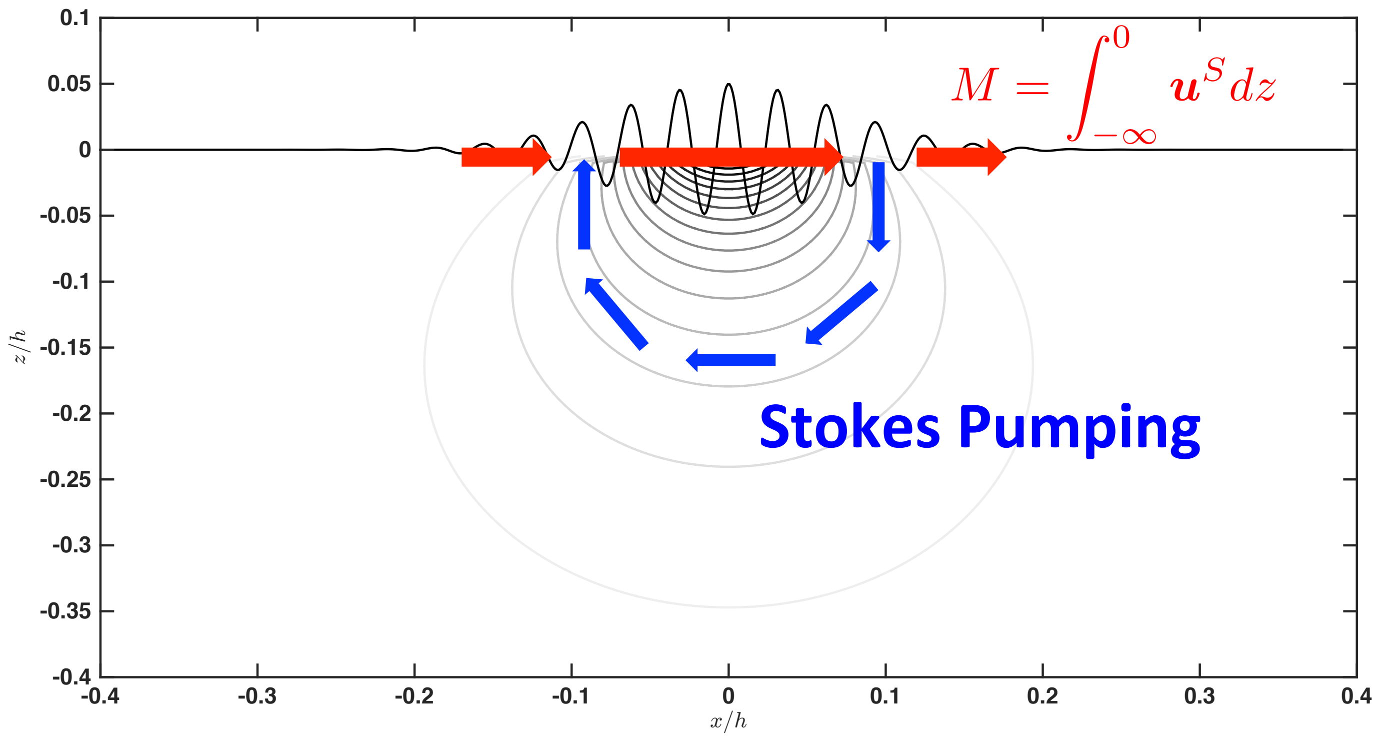
$$@z = 0, \quad \overline{w} = M_x$$

$$M = \overline{u_1 \zeta_1} = \frac{1}{2} ck|a|^2$$

$$= \int_{-\infty}^{0} \boldsymbol{u}^S dz$$

What Creates the Return Flow?

- Horizontally varying waves (Stokes drift) move water along at different rates creating convergence/divergence: Stokes pumping.



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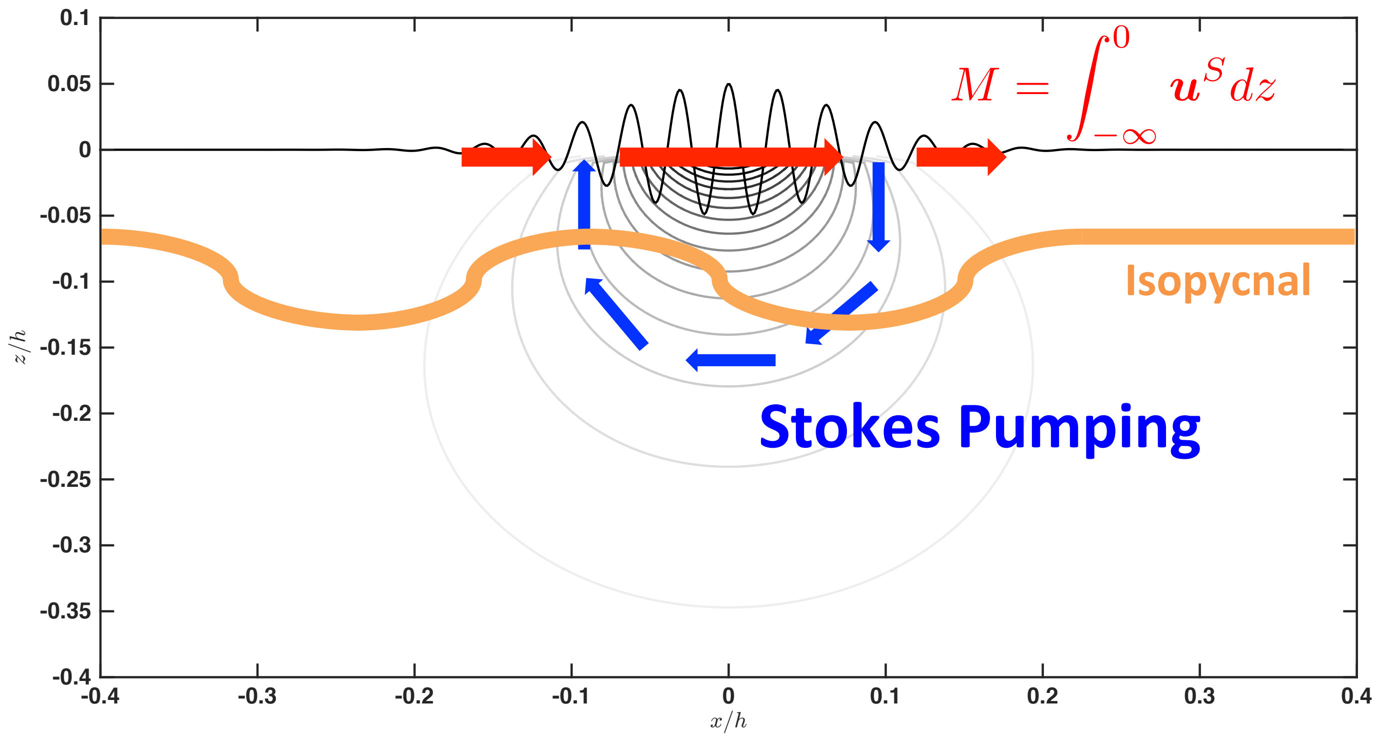

The Stratified Return Flow in 2D

$$\overline{u}_2 = -\psi_z, \quad \overline{w}_2 = \psi_x$$

O(ε²) Assumption: 2D flow

Advantage in 2D: N(z)

$$\psi_{zt} = \left(\overline{p}_2 + \frac{1}{2}\overline{|\boldsymbol{u}_1|^2}\right)_x$$

$$\psi_{xt} = -\left(\overline{p}_2 + \frac{1}{2}\overline{|\boldsymbol{u}_1|^2}\right)_z + \overline{b}_2$$

$$\overline{b}_{2t} = -\psi_x N^2$$

Buoyancy is back at this order

Identically zero for irrotational flow. Small outside the mixed layer. Includes terms like Stokes Vortex, Stokes Coriolis, etc.

$$\left[\partial_t^2\left(\partial_x^2 + \partial_z^2\right) + f^2\partial_z^2 + N^2\partial_x^2\right]\psi = A(x,y,t)e^{2kz}$$

Negligible. Consider the packet $Ro = \dfrac{c}{2f\ell} \sim 200$

BC: $@z = 0, \quad \overline{w} = M_x \quad \rightarrow \quad @z = 0, \quad \psi = M$

The Stream Function in the Group Frame

Now we will move into the frame moving at the group velocity (c/2)

$$\tilde{x} = x - \frac{1}{2}ct \qquad \partial_t \to -\frac{1}{2}c\partial_{\tilde{x}}$$

$$\left[\partial_{\tilde{x}}^2 + \partial_z^2 + \left(\frac{2N}{c}\right)^2\right]\psi = 0$$

Exercise: assume constant stratification (N), and ψ is the stream function for an internal wave with horizontal wavenumber q, and vertical wavenumber m. Then we have a restriction on the internal wave phase speed:

$$-q^2 - m^2 + \left(\frac{2N}{c}\right) = 0$$

IW ***phase*** speed $\qquad \dfrac{N}{\sqrt{q^2 + m^2}} = \dfrac{c}{2} \qquad$ SW ***group*** (forcing) speed

How Stratified Must the Ocean Be to Allow 2D IW Generation?

Again assuming constant stratification and vertical modes $\phi_n(z)$ we can put a lower bound on the stratification required to for IW generation.

$$\left[\frac{d^2}{dz^2} + \left(\frac{2N}{c}\right)^2 - q_n^2\right]\phi_n = 0$$

How Stratified Must the Ocean Be to Allow 2D IW Generation?

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Can solve for any N(z)

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Assume constant N for now

$$\left[\frac{d^2}{dz^2} + \left(\frac{2N}{c}\right)^2 - q_n^2\right]\phi_n = 0$$

substitute

$$\phi_n = \sin\left(\frac{n\pi z}{h}\right)$$

$$-\left(\frac{n\pi}{h}\right)^2 + \left(\frac{2N}{c}\right)^2 + q_n^2 = 0$$

$$Nh > \frac{\pi c}{2}$$

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Looks like

$$\int_{-h}^{0} N dz = \frac{n\pi c}{2}$$

The stratification in the ocean turns out to be a little TOO WEAK for IW emission under this 2D assumption in the finite depth case. SEVERAL full depth profiles were tested and none were sufficiently stratified.

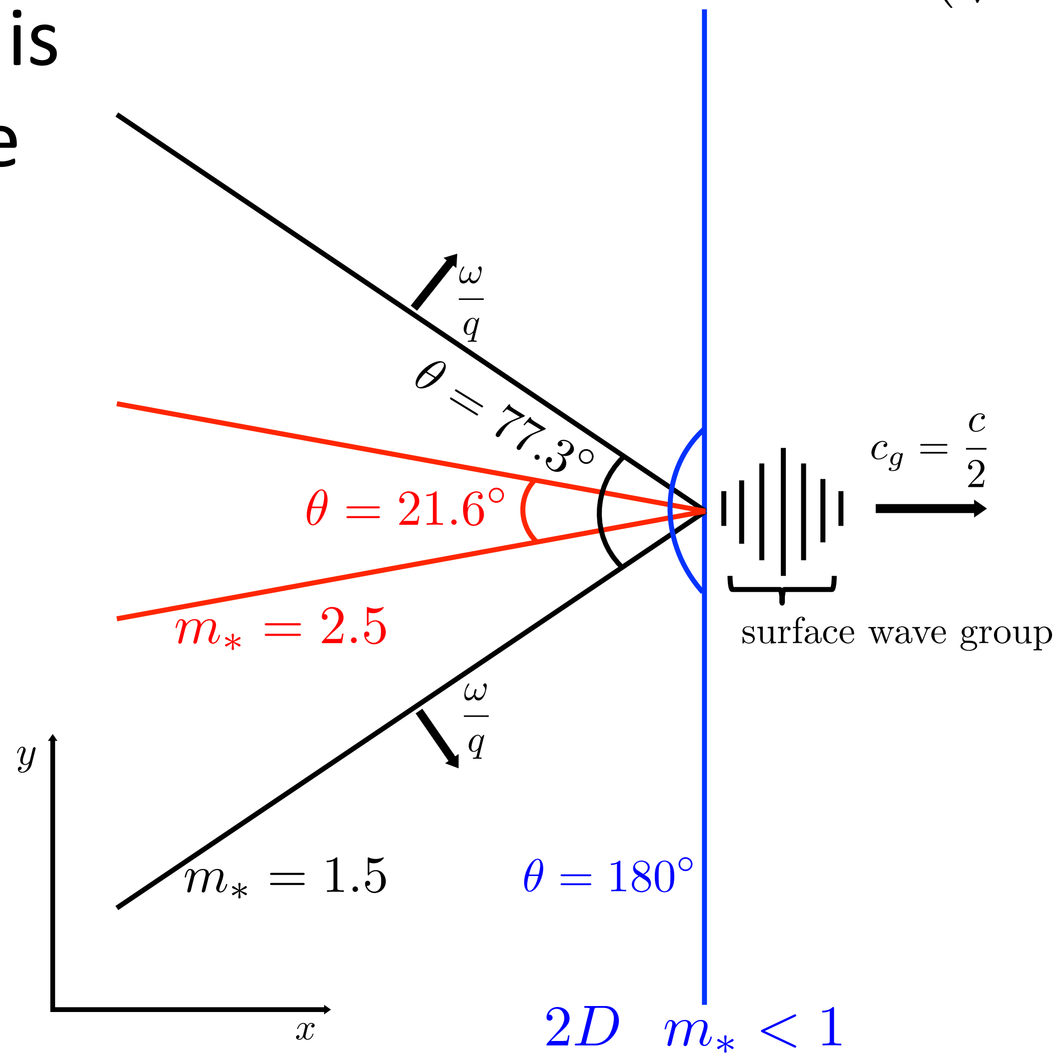
3D Internal Wave Wakes

- In 2D the IWs are TOO SLOW (because the stratification is TOO WEAK) to keep up with their energy source (SWs), but in 3D, they need not be as fast.
- IWs propagating obliquely to the SWs may still extract energy from the waves.
- Assumptions/downsides (relative to 2D method): Constant N.

The Wake Angle

$$m_* = \frac{n\pi c}{2hN} \qquad \theta = 2\arctan\left(\frac{1}{\sqrt{m_*^2 - 1}}\right)$$

- The wake angle is predicted by the stratification, depth, and SW group speed.



Energy Flux

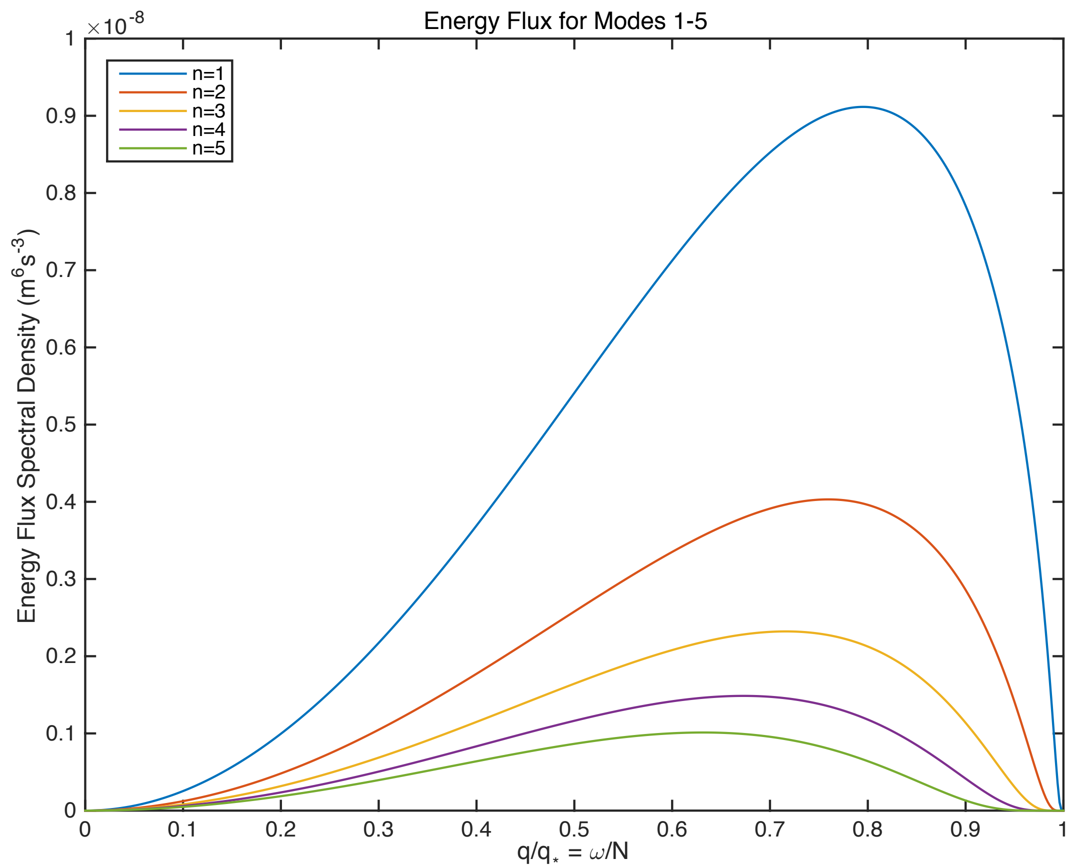
$$J = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varpi_2(x,y,0,t) w_2(x,y,0,t)\,dxdy$$

$$J_n = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{\varpi}_2 \hat{w}_2^* \frac{dqds}{2\pi} = \frac{c}{2\pi h}\int_0^{\infty} q^2 |\hat{M}|^2 \sqrt{\frac{q_*^2 - q^2}{m_n^2 - q_*^2 + q^2}}\,dq$$

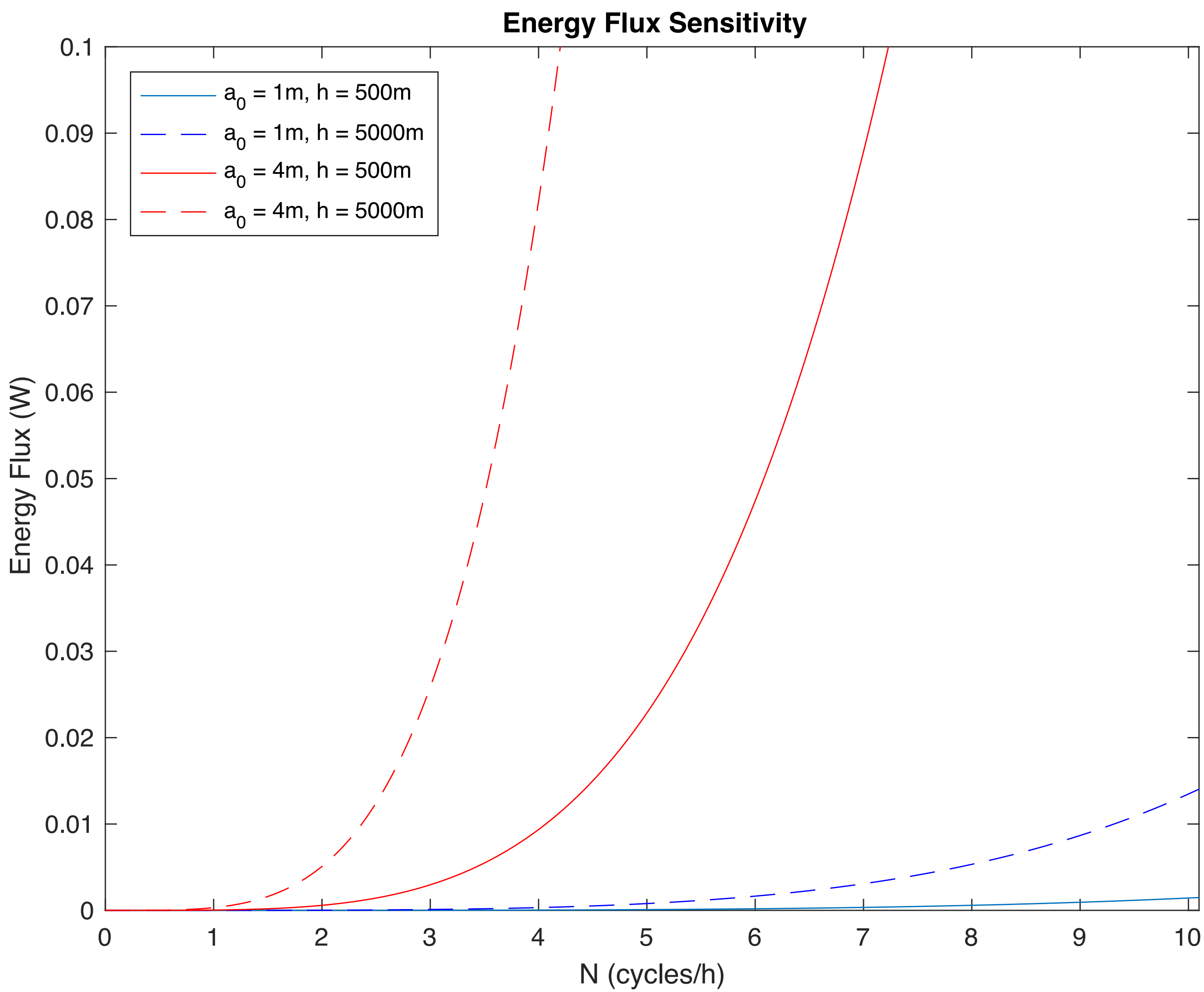
- Energy flux peaks with IW frequency close to N.
- Total energy flux in all modes:

$$J \approx 5 \times 10^{-3} W$$

(per wave group)



Energy Flux is Very Sensitive to Stratification, Depth, and SW Height



How Much Energy Are We Talking About?

- **Upper bound:** SW loss of energy to IW cannot be enough to kill SWs. SW half life: >1000 years.
- **Lower bound:** SW to IW energy flux should be seen in observations. Pinkel (1975) show peak in near N displacement (by IWs) spectra.

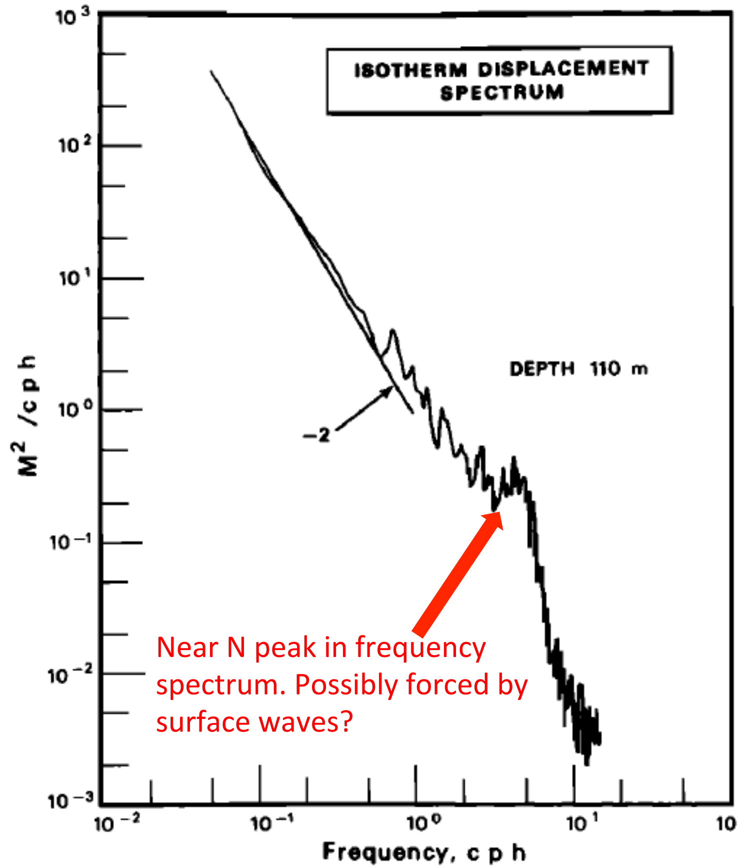


Fig. 6. Hawaii isotherm displacement spectrum, 40 d.f., mean depth 110 m.

Pinkel, JGR 1975

$$\frac{E_{obs}}{J_{theory}} = \text{Timescale of forcing near N peak}$$

$$= 1 \text{ day} - \text{ years (depending on N, h, and } a_0\text{)}$$

Conclusions

- In 2D, the IW phase speed must match the SW group speed to for IW generation.
  - The stratification in most of the open ocean is too weak to produce such fast IWs.
  - The stratification is still too weak even considering non-uniform stratification with sharp pycnoclines.
  - Despite this, even in the absence of IW radiation, the return flow is altered significantly from the dipole shape shown by e.g. McIntyre (1981).
- In 3D IWs are always generated (even in very weak stratification), and propagate at oblique angles to the SWs given by the stratification, depth and SW group speed.
- The energy lost by the surface waves is sufficiently small that swell can propagate long distances without substantial damping (half life ~ 1000y).
- The energy input to surface waves may contribute significantly to observed near N IW energy.